
J. R. Wolf and the Zürich Sunspot Relative Numbers

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Introduction

Sunspot sightings and their subsequent recordings have long enjoyed a fascinating history. Prior to the beginning of the seventeenth century, all sightings were by naked eye. When Lippershey produced the first really commercial telescope in 1608, and, a few years later, when Galileo improved upon its basic design, the history of sunspot sightings became inextricably bound up with the development and utilization of this remarkable instrument. Three men quickly claimed priority in identifying the true nature of sunspots: Galileo in Venice, Johannes Goldsmid ("Fabricius") of the Netherlands, and Christoph Scheiner of Rome. Their observations showed that sunspots were contiguous to the surface of the sun, they were carried around by the sun's rotation, they frequently occurred in groups, and within a single group the spots moved relatively to each other. From series of sunspot sketches the sun was determined to have a rotation period of about 27 days. Although sunspot viewing became a very popular pastime, no long series of regular observations were made, and the 70 years from 1645 to 1715, when hardly any spots were observed on the sun's disk, only served to frustrate those whose curiosity had been aroused regarding unusual solar activity. By the close of the eighteenth century, general scientific opinion on sunspots was that they "observe no regularity in their shape, magnitude, number, or in the time of their appearance or continuance" (as quoted in a 1764 scientific textbook). The important breakthrough came in 1838 when Samuel Heinrich Schwabe published twelve years of his own sunspot observations; in [9], he listed for each year the number of days he had observed the sun, the average number of sunspot groups, and the number of days in which the sun was spotless. Clearly, the more numerous the spotless days, the lower the sunspot count. Schwabe published further annual sunspot data until 1844, when, for the first time, he recognized a "certain periodicity" in his 18 years of sightings, and he judged the sunspot period to be of "about 10 years" [10]. Sur-

prisingly, Schwabe's remarkable announcement evoked almost no attention within the scientific community. It was not until the painstaking research of Rudolf Wolf, the central figure of this essay, that the sunspot cycle was determined to be of the order of 11 years rather than 10 years, and that the far-reaching relationship between sunspot activity and geomagnetic activity on Earth was first discovered. Wolf has not yet received the scientific recognition that he deserves, and, in fact, in the statistical literature his work has been consistently confused with Alfred Wolfer, a student of his [4].

Johann Rudolf Wolf

We turn first to a brief biographical sketch of Wolf. Born on 7 July 1816 at Fallanden, a small village near Zürich in Switzerland, he was the youngest of three, his father being the village pastor. After his father died in 1827, his mother and the children moved to Zürich. From 1828 to 1833, Rudolf's aptitude for mathematics blossomed under teachers such as J. C. Horner and

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K. H. Graffe, and in 1833 he became a student at the newly founded University of Zürich, where he studied higher mathematics under J. L. Raabe. Wolf then spent the three years 1836 to 1839 travelling through Europe and attending lectures at some of the finest intellectual centers there. At the University of Vienna, he studied astronomy and physics for two years until 1838, when he moved to Berlin and attended lectures at the University by Encke, Dirichlet, Poggendorff, Steiner, and Crelle. His stay in Berlin was a huge success and resulted in numerous personal contacts. In August 1838 Wolf travelled to Göttingen, where, armed with a recommendation from Encke, he went to the university there to see C. F. Gauss. Wolf wrote in his diary of the 5 August meeting: "He gave me a friendly reception, much greater than I had expected. He is a grey 60-year-old with fiery eyes, yet neither his figure nor his features are particularly remarkable." Among other things, Wolf and Gauss spoke about establishing a magnetic observatory in Zürich, partly because of the higher altitude, and partly because of its location between München and Milan where such observatories already existed. It is clear that Wolf was fascinated by the work Gauss was doing to set out a general theory of geomagnetism, and from comments in his diary of that meeting Wolf showed that he was well versed in the latest developments in this field. From Göttingen, Wolf travelled to Bonn, then Brussels, and finally to Paris, where he attended lectures at the Sorbonne by Poisson (on mechanics) and by Libri (on the calculus of probability, including the uncertainty of information in actuarial tables, and on constant loss in games of chance).

As soon as he returned to Zürich at the end of 1838 he applied unsuccessfully for a Privatdozent position at the University of Berne, and then accepted a position at the Berne Realschule as a mathematics and physics teacher. His first book, *The Theory of Linear Mappings in the Plane*, appeared in 1841. In 1844, he also began lecturing, unpaid, on mathematics and astronomy at the University of Berne, and, because of internal politics, instead of promoting him there to a mathematics professorship, he was appointed, in the spring of 1847, to a salaried position as Dozent and, as compensation, to the Directorship of the Berne Observatory.

During the early 1840's, his publications reflected his mathematical interests, with papers on prime number theory and geometry. Later, during the years 1849–52, he wrote a number of lengthy papers on probability and statistics; specifically, discussions on least squares (possibly influenced by a lecture given by Dirichlet that he had attended at the University of Berlin), trimmed means (making him a very early writer on the general area of robust statistical estimation), and the relation between theoretical probability distributions and their empirical counterparts. One paper, in particular, ex-

plains the solution to Buffon's Needle Problem at great length, and in a follow-up paper, he inverts that solution to provide a method for estimating π ; his estimation procedure was pure Monte Carlo, and he is now credited in [3] as being the first to estimate π using such a technique. He continued publishing articles on the laws of probability up to the very last years of his life, and took great delight in explaining at length reasons behind some of the well-known paradoxes of the subject.

In 1852, Wolf received an honorary doctorate from the University of Berne for his fundamental work on sunspot periodicity. The next year he was appointed Ausserordentlicher Professor of Mathematics, and, owing to his important discovery of the relationship between sunspots and terrestrial magnetism, he was quickly promoted to Professor of Mathematics. In April 1855, the Gymnasium in Zürich named Wolf to be Raabe's successor as Professor of Mathematics, and so he moved back to Zürich, where he was also appointed Professor of Astronomy at the Eidgenössische Technische Hochschule (E.T.H.). When the Swiss Federal Government started construction of a new Swiss Federal Observatory in Zürich, Wolf was their obvious choice as its first Director, and accordingly he resigned his Chair at the Gymnasium.

During his lifetime, Wolf founded two major scientific journals in Switzerland, both of which still exist today. In 1843, he founded the *Mittheilungen der Naturforschenden Gesellschaft in Berne*, and in 1856, as soon as he returned to Zürich, he founded the *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*. Both journals published extensive historical notes on Swiss mathematicians and physicists. Wolf was a prolific writer, who published hundreds of articles on astronomy, over fifteen books (not counting revised editions) on the history of science and biographies of men of science, and carried out the editorial duties for his two journals. On 11 November 1864, he was elected a Foreign Associate of the Royal Astronomical Society; in 1885, he was elected a member of the Paris Academy; and, in 1893, he was made an honorary member of the Austrian Meteorological Society. In his last years of life, his lectures were primarily of historical content. He died on 6 December 1893 aged 77 years old. A complete bibliography of Wolf may be found in the *Catalogue of Scientific Papers*, published by the Royal Society of London. The *Dictionary of Scientific Biography* gives some biographical information on Wolf (by J. J. Burckhardt), as does the *Encyclopaedia Britannica* and *World Who's Who in Science*.

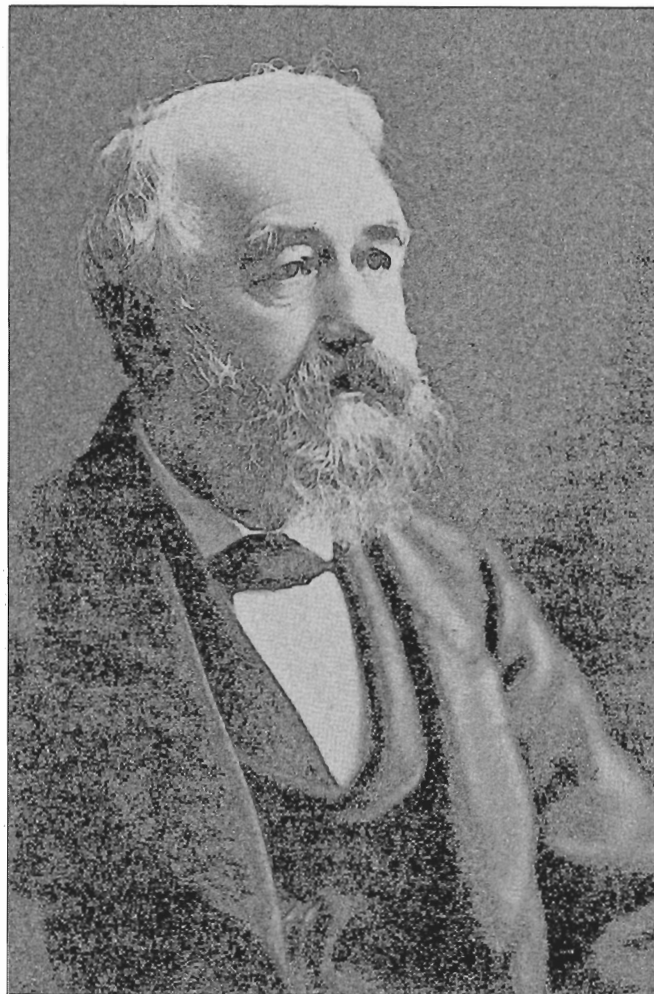
Wolf's Statistical Work on Sunspots

Wolf became Director of the Berne Observatory in 1847 just prior to a sunspot maximum, and immediately he became fascinated by reports of the "rather numerous

sunspots" being observed at that time. In December 1847, after corresponding with other sunspot observers, he began his own viewing, and spent the entire year of 1848 just familiarizing himself with observational techniques. Initially, his published reports described only the larger sunspots together with hand-drawn pictures. Starting in January 1849, he systematically recorded his own series of sunspot observations, the first of which was published in August that year. Building on the work of Schwabe, he provided the most extensive documentation of sunspot behavior that had ever been published. Whereas Schwabe had published annual (and, later, monthly) data, Wolf now set out his daily records for those days that were spent in observing the sun. Specifically, he recorded each day (1) the total number, g , of visible groups, including isolated spots, (2) the total number, f , of individual spots within those groups, (3) the sky conditions, which he classified as either "free of clouds", "spots could be seen through clouds", or "spots could not be seen at all", and (4) which of two telescopes, large or small, he used for his viewing. These daily records were published twice each year from 1849 to 1855.

The next step was to be a crucial one for Wolf. From Schwabe's published comments and his own observations he realized that the average number of sunspot groups (computed annually or monthly as Schwabe had done) actually gave a poor summary of sunspot activity. To devise a more sensitive measure, Wolf adjusted the number of sunspot groups observed each day by adding to it one-tenth the number of individual spots for that day, yielding a daily *Relativzahlen* or "relative number" of $g + (1/10)f$; to obtain a monthly relative number, he then averaged the appropriate daily relative numbers by dividing their sum by the number of days, n , he observed that month [13]. It is important to note that the three numbers, g , f , and n , were taken for only those days in which *both* the larger telescope was used and the sky was "free of clouds"; all additional observations were considered—at that time—extraneous, and were ignored in the final computations. Later, he would relax these constraints. It is natural to ask here why Wolf chose $1/10$ as the coefficient of f . From his few remarks and comments on this point, it seems clear that he had tried other factors, but was uncomfortable with any of the alternatives; also, he felt that 10 was a simpler number to deal with in computations than, say, 11 or 12. In his own words, "I chose the number 10, on the one hand, because it seemed to work well for a large number of instances which I studied for this purpose, and, on the other hand, because it was simpler to deal with than any other number close to it" [16].

Schwabe then provided the impetus for Wolf's two major scientific discoveries. In the third volume of his huge treatise *Cosmos*, published in 1851, Alexander von Humboldt included 25 years of Schwabe's data



Johann Rudolf Wolf, courtesy of the Library of the Eidgenössische Technische Hochschule Zürich.

(from 1826 to 1850), and quoted Schwabe's views on a 10-year sunspot cycle, and on a conjecture that such periodicity might even be seen in older observations than his. Wolf immediately set out to verify Schwabe's conjecture by searching through several hundred volumes in the libraries of Basel, Berne, and Zürich for old sunspot records. One unpublished manuscript—dated 1776—he discovered was written by the Danish scientist Christian Horrebrow (1718–1776), who suggested the likelihood of a periodic variation in sunspots:

Even though it follows from observations that changes and variations in sunspots are frequent, one cannot find any definite rule by which the order and duration of this variation is completed. The main reason for this is that astronomers have made very few attempts so far at frequent observations of sunspots, undoubtedly because they felt that nothing of any interest for astronomy or physics would come of it. However, it is hoped that by means of frequent observation one will also find a period here similar to the movements of other heavenly bodies.

The stage was set when John Lamont, Director of the Munich Observatory, announced on 23 December

1851 that magnetic observations taken by himself at Göttingen during 1835–41 and at Munich during 1841–50 on annual mean diurnal variations of magnetic declination (compass-bearing of magnetic north from geographical north) indicated a periodicity of $10^{1/3}$ years [6]. Wolf quickly recognised that the epochs of maxima and minima of the variations in magnetic declination (as discovered by Lamont) corresponded exactly with those of sunspot activity (as discovered by Schwabe), but thought little of his findings. It took another sunspot observer, Julius Schmidt of Bonn, to assure him that the correspondence was not only new but important. Wolf communicated his finding to the Zürich Society and published it on 31 July 1852 [14], concluding that he considered himself "lucky to have realized these coincidences and thus to have become the discoverer of an important natural law". Wolf, however, was not the only person to have noticed the correspondence between the two phenomena, and, furthermore, neither was he the first. Edward Sabine, one of the leading figures in British geomagnetic research, had arrived at similar conclusions that same year, presenting his findings to the Royal Astronomical Society on 18 March 1852 and reading them on 6 May 1852 [8], and, in a paper dated 17 June 1852 and published in August 1852, Alfred Gautier, Director of the Geneva Observatory, and one of the few scientists who recognised the importance of Schwabe's work back in 1844, also linked Schwabe's sunspot periodicity to Lamont's findings [2]. All three men, Sabine, Wolf, and Gautier, arrived at the same conclusions quite separately and independently.

Using this discovery plus Schwabe's conjecture as motivation, Wolf wrote an extremely lengthy and elaborate article in November 1852 entitled "New investigations into the sunspot period and its meaning" [15], in which all his research towards a solid historical foundation of sunspot periodicity was carefully set out and discussed in the greatest detail. In that paper, he established "the exact duration of the sunspot period", and his approach was quite straightforward. If sunspot activity is indeed periodic, then the difference between any two dates of peak activity should be an integer multiple of the duration of the period. A similar argument would also hold for the difference between two dates of extremely low sunspot activity. If a sufficient number of peaks and valleys could be identified, perhaps allowing for error in the observational assessment of those peaks and valleys, then an appropriate averaging of those differences should provide a reasonable estimate of the duration of the period. Using this type of argument, Wolf settled on six maxima

$$\begin{array}{ll} (M1) 1626.0 \pm 1.0, & (M2) 1717.5 \pm 1.0, \\ (M3) 1816.3 \pm 1.0, & (M4) 1829.5 \pm 1.0, \\ (M5) 1837.5 \pm 0.5, & (M6) 1848.6 \pm 0.5, \end{array}$$

and six minima

$$\begin{array}{ll} (m1) 1645.0 \pm 1.0, & (m2) 1755.5 \pm 0.5, \\ (m3) 1810.5 \pm 1.0, & (m4) 1823.2 \pm 0.5, \\ (m5) 1833.6 \pm 0.5, & (m6) 1844.0 \pm 0.5, \end{array}$$

in which he felt most confidence. Then, he computed 16 estimates of the sunspot period as follows:

$$\begin{array}{l} M3 - M2 = 9(10.98 \pm 0.16) \\ M4 - M2 = 10(11.20 \pm 0.14) \\ M5 - M2 = 11(10.91 \pm 0.10) \\ M6 - M2 = 12(10.93 \pm 0.09) \end{array}$$

$$\begin{array}{l} M3 - M1 = 17(11.19 \pm 0.08) \\ M4 - M1 = 18(11.31 \pm 0.08) \\ M5 - M1 = 19(11.13 \pm 0.06) \\ M6 - M1 = 20(11.13 \pm 0.06) \end{array}$$

$$\begin{array}{l} m3 - m2 = 5(11.00 \pm 0.22) \\ m4 - m2 = 6(11.28 \pm 0.12) \\ m5 - m2 = 7(11.28 \pm 0.10) \\ m6 - m2 = 8(11.06 \pm 0.09) \end{array}$$

$$\begin{array}{l} m3 - m1 = 15(11.03 \pm 0.09) \\ m4 - m1 = 16(11.14 \pm 0.07) \\ m5 - m1 = 17(11.09 \pm 0.07) \\ m6 - m1 = 18(11.06 \pm 0.06). \end{array}$$

Each expression has the general form $\xi_i \pm \delta_i = c_i (p_i \pm u_i)$, where c_i is an integer estimate of the number of complete cycles in ξ_i years, each cycle lasting p_i years plus-or-minus u_i years (a "probable error" in assessing p_i). Wolf's estimate of the true sunspot period, p , say, was a weighted average of the sixteen p_i values, where p_i had weight $w_i = 1/u_i^2$:

$$\hat{p} = (\sum_{i=1}^{16} w_i p_i) / (\sum_{i=1}^{16} w_i) = 11.11 \text{ years.}$$

This estimate has held up remarkably well over time [1]. A few comments on Wolf's procedure are worth making. Notice that the above factorizations each have p_i -values of around 11; although no theoretical explanation was given in the paper to justify why p_i -values should be around 11 rather than, say, 10 or 12, Wolf set out in the second part of the paper an empirical justification based on the historical sunspot records he has collected. The above factorizations also show that two maxima (or minima) far apart tend to yield a smaller value of u_i than two close maxima (or minima), a questionable assumption to make. A more serious criticism relates to the fact that the p_i -values are derived from overlapping time periods, and hence are correlated; yet Wolf's formula implicitly treats them as if they were independent estimates of the true sunspot period p . Despite these criticisms, however, Wolf's result that the sunspot period was, on average, just over

11 years rather than 10 years was a remarkable achievement and, for the scientific community, quite unexpected. Wolf also appended a plus-or-minus term of 0.038 years (or 13.87 days) to the 11.11-year period, but never explained how he arrived at that number. He then rounded out his paper by further demonstrating that his 11.11-year sunspot period gave a closer fit to Lamont's magnetic declination data than did Lamont's own $10^{(1/3)}$ -year period. This last result was to make Lamont and Wolf antagonists for life.

In 1855, after moving to Zurich, Wolf redefined his daily sunspot number. By multiplying his 1851 definition by 10, it now became $10g + f$ [16]; this he considered to be an approximation to a solar quantity which he knew was important, but yet could not measure:

To be sure, instead of the relative number obtained in this fashion, I would have preferred numbers which were proportional to the total area of the spots, but it would have been too much effort to make the necessary measurements and calculations; for that purpose, not only would I have had to measure the area of the spots, but also to determine their exact location on the surface of the sun in order to reduce the dimensions which appear to be foreshortened.

Actually, the approximation is quite good; see p. 238 of [12]. Wolf went still further. To plug up the numerous gaps in his own data base, he, first, incorporated Schwabe's observations where possible, assigning them equal weight with his own, and, second, he relaxed his 1851 definition so that all his data taken on days in which "spots could be seen through clouds" could now be included, as well as all data recorded with the smaller telescope. From this expanded data base, he recomputed monthly and yearly mean sunspot relative numbers for the period 1849–1855, and published tables of daily sunspot numbers and their monthly and yearly averages on an annual basis from 1856. In 1859, he published a revised list of epochs of maxima and minima from the time of the discovery of the telescope, and noted that while the mean sunspot period differed little from 11.11 years, considerable fluctuations around that number were the rule rather than the exception.

Then, in 1861, Wolf changed the definition of sunspot relative number a third time, by using a *Reduction-factor* ("reduction factor") A as a variable multiplier whose value depended on the observer, his telescope, and the viewing conditions; the sunspot relative number now became $A(10g + f)$, and Wolf himself assigned the value of A as appropriate. To his own records, Wolf set $A = 1$ when he used the large telescope, and $A = 1.5$ when he used the small telescope. The first time these reduction factors were applied, they appeared in 1861 for the daily data of the year 1860 [19]. In 1867, he published an extensive list of reduction factors that he had assessed for the historical sunspot records.

In 1875, Wolf published a smoothed version of the sunspot relative numbers for 1836–73, by first computing a moving average of length 12 over the monthly series, and then computing a second moving average, this time of length two, over the previously smoothed series; this is equivalent to the formula

$$Sm(r_t) = \{r_{t-6} + 2 \sum_{k=-5}^5 r_{t-k} + r_{t+6}\}/24,$$

where $\{r_t\}$ is the series of monthly mean sunspot numbers. This method is still used today. It also appears to be one of the first instances in which an empirical time series was smoothed by a repeated smoothing operation. By 1876, Wolf's listings of sunspot relative numbers are almost identical to those appearing in [12]. From then on, all that was left to do was to improve the series through additional data sources. For some cases, these sources filled gaps in the series, while for others they confirmed or modified existing data. From 1877, Wolf started a new policy of assigning values of the reduction factor A for each observer more frequently than once a year. The sunspot numbers remained in this form until Wolf died at the end of 1893, but his work was carried on by his successors, Alfred Wolfer, W. Brunner, and M. Waldmeier.

Scientific Opposition to Wolf's Discoveries

The determination of an 11.11-year sunspot period (instead of the ten-year period of Schwabe) and also of its alleged fit to Lamont's data on magnetic declination (instead of the $10^{(1/3)}$ -year period of Lamont) caused many in the scientific community to oppose Wolf's findings. As a result, Wolf spent a great deal of time arguing his case to critics through articles and personal letters, yet he remarked, in amazement, that [16]

There are still astronomers and authors who deal with this subject and who refuse to believe the results of my investigations or who simply ignore them completely.

In another paper [17], he emphasized the need for "making possible the faith that is lacking among a few of the astronomers", and to some extent even regarded such controversy as a challenge.

One such sceptic was the astronomer Manuel J. Johnson, President of the Royal Astronomical Society. On 13 February 1857, Schwabe was awarded the Gold Medal of the Society, and Johnson remarked in his Presidential Address that although the older astronomers had found nothing definite in the appearance of sunspots, Schwabe had brought order into the apparent chaos through his long years of observation; he then added [5]:

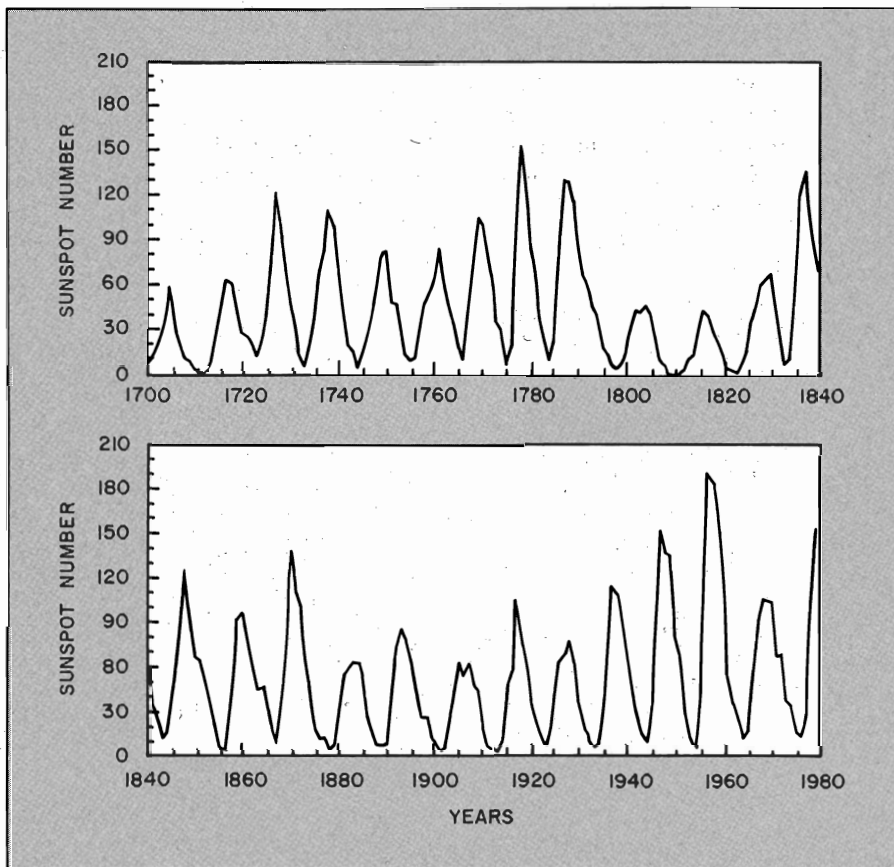


Figure 1. Annual mean Zürich sunspot relative numbers, 1700–1980.

But we are naturally led to inquire whether we can connect them with the statements of former observers. Professor Wolf, who has entered deeply into the general question, published a few years ago . . . all the notices he was able to find, scattered in books and memoirs, from the time of the discovery of the spots in 1611; and from a comparison of them with more modern determinations he deduced a period of 11.1 years. Without depreciating the industry and ingenuity displayed in that paper, I confess I am not disposed to place much reliance on the result, owing to the insufficient data on which it appears to me to be founded. The inquiry is rendered very difficult, because the old observers attended much more to the physical characteristics of the spots, and to their time of rotation, than to their number.

Wolf's answer to such charges was to keep publishing further historical proof of his findings.

One of Wolf's accomplishments in this direction was his rediscovery of Thomas Harriot's 199 sunspot observations dating from 11 December 1611 to 28 January 1613, which lay buried for almost two centuries and were thought lost. Not wanting to leave any stone unturned to prove his point, Wolf wrote to Richard Carrington in England asking for help in extracting "the desired treasure". Carrington travelled to Petworth House in Sussex, found all Harriot's drawings, copied them and the accompanying notes, and sent them off to Wolf together with a summary table showing Harriot's observations in the same form as Wolf had published his own data. Wolf became excited when he saw

Harriot's data, and felt that they were much more important than the data of Fabricius. Wolf went on to demonstrate that Harriot's data was completely consistent with his 11.11-year sunspot period, and concluded that [18].

I would be very surprised if there were any astronomers and physicists who, after this new confirmation of my period, still do not believe me.

Yet he continued to publish his notes on sunspot sightings, both ancient and current. Between 1857 and 1874, Wolf published a total of 324 of these notes, although no further historical material was published after 1875.

Lamont remained firmly unconvinced by Wolf's endeavours, especially since it disagreed completely with his own findings. This animosity led Wolf and himself to write a number of articles in which each went to great lengths to stubbornly reject the claims of the other, showing that if one blindly followed the period set down by the other it would only lead to major contradictions. In the end, both decided that it was quite impossible to convince the other of the legitimacy of their respective claims.

Not all current opinion was against Wolf. Baronet John Herschel, for example, in a letter to Wolf dated 21 April 1867, wrote that Wolf's results formed "a most important and valuable work which merits the grati-

tude of every astronomer". Another who was convinced by Wolf's calculations was Schwabe, who later adopted Wolf's 11-year sunspot period as fact.

Concluding Remarks

The Zürich sunspot relative numbers are listed in [12] up to 1960; more recent data may be found in the *Quarterly Bulletin on Solar Research*, the *Journal of Geophysical Research*, and the amateur astronomy journal *Sky and Telescope*. The annual means of the sunspot numbers from 1700 to 1980 are displayed in Figure 1, the cyclic nature of the series being quite evident. From January 1981, following the retirement of Max Waldmeier, the fourth Director of the Swiss Federal Observatory, responsibility for sunspot research was transferred from Zürich to the Sunspot Index Data Centre in Brussels, Belgium.

We have shown in this essay that Rudolf Wolf discovered the 11-year sunspot cycle, and was so convinced of the validity of his finding that he fought the sceptics and all those who questioned his methods. He was a co-discoverer of the important relationship between sunspots and geomagnetic activity on Earth, although the real connection between these two phenomena would be realized much later when sunspots were found to possess intense magnetic fields of their own. He devised an index of solar activity, the sunspot relative numbers, that is still in use today. Unfortunately, no really convincing theory, either astronomical or statistical, has been demonstrated to explain sunspot behavior over time, and no known method of making long-term forecasts of solar activity is currently available [11].

Sunspots remain an enigma to scientists. To statisticians, they offer a challenge: the sunspot numbers have been shown to contain idiosyncracies that suggest, quite strongly, that the underlying statistical mechanism by which they are generated is nonlinear, nonstationary, and non-Gaussian, and as such they are used primarily as a yardstick to compare and judge new statistical modelling and forecasting methods. The reader may wish to consult [1] and [7] for further details.

One final comment: although Wolf thought of himself primarily as an astronomer, with deep roots in the history of the subject, his lifelong interest in probability and statistics was substantial; he will be remembered for (at least) three statistical contributions: his Monte Carlo estimation of π , his smoothing of empirical time series, and, of course, the sunspot numbers.

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The profound study of nature is the most fertile source of mathematical discoveries.

Fourier