Key Terms and Concepts for Chapter 6

Vector Space – 10 properties

Closure
Linear Combinations
Span
Linearly Independent
Linearly Dependent

R_n, M_{mn}, P_n, P, F[a,b]

A **real vector space** is a set of elements V together with two operations $\oplus$ and $\odot$ satisfying the following properties:

(α) If $\mathbf{u}$ and $\mathbf{v}$ are any elements of V, then $\mathbf{u} \oplus \mathbf{v}$ is in V (i.e., V is closed under the operation $\oplus$).

(a) $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$, for $\mathbf{u}$ and $\mathbf{v}$ in V.

(b) $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$, for $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in V.

(c) There is an element $\mathbf{0}$ in V such that

$$\mathbf{u} \oplus \mathbf{0} = \mathbf{0} \oplus \mathbf{u} = \mathbf{u}, \quad \text{for all } \mathbf{u} \text{ in } V.$$

(d) For each $\mathbf{u}$ in V, there is an element $-\mathbf{u}$ in V such that

$$\mathbf{u} \oplus -\mathbf{u} = \mathbf{0}.$$

(β) If $\mathbf{u}$ is any element of V and c is any real number, then $c \odot \mathbf{u}$ is in V (i.e., V is closed under the operation $\odot$).

(e) $c \odot (\mathbf{u} \oplus \mathbf{v}) = c \odot \mathbf{u} \oplus c \odot \mathbf{v}$, for all real numbers c and all $\mathbf{u}$ and $\mathbf{v}$ in V.

(f) $(c + d) \odot \mathbf{u} = c \odot \mathbf{u} \oplus d \odot \mathbf{u}$, for all real numbers c and d, and all $\mathbf{u}$ in V.

(g) $c \odot (d \odot \mathbf{u}) = (cd) \odot \mathbf{u}$, for all real numbers c and d and all $\mathbf{u}$ in V.

(h) $1 \odot \mathbf{u} = \mathbf{u}$, for all $\mathbf{u}$ in V.

The elements of V are called **vectors**; the real numbers are called **scalars**. The operation $\oplus$ is called **vector addition**; the operation $\odot$ is called **scalar multiplication**.

Properties shared by all vector spaces:

- **A "zero" element or additive identity, denoted by 0; <u>WARNING</u>: for certain definitions of addition & scalar multiplication the "zero" element may not involve numerical zeros**

If V is a vector space, then:

(a) $0\mathbf{u} = \mathbf{0}$, *for every* $\mathbf{u}$ *in* V.

(b) $c\mathbf{0} = \mathbf{0}$, *for every scalar* c.

(c) *If* $c\mathbf{u} = \mathbf{0}$, *then* $c = 0$ *or* $\mathbf{u} = \mathbf{0}$.

(d) $(-1)\mathbf{u} = -\mathbf{u}$, *for every* $\mathbf{u}$ *in* V.

- **Subspace**

Particular subspaces: <u>Zero subspace</u>, <u>Row space</u> of matrix A denoted by row(A), <u>Column space</u> of matrix A denoted by col(A), <u>null space or solution space</u> of matrix A denoted by ns(A)

- **Basis for a vector space**
  Natural basis for certain vector spaces
- **Dimension of a vector space or a subspace**
- **Rank of a matrix**
- **Orthogonal sets in R^n**
- **Orthonormal sets in R^n**