EIGEN SUMMARY and Related Information Fall 2009

I. If λ is an eigenvalue of **A** with corresponding eigenvector **x**, then k**x,** for any scalar $k \neq 0$, is also an eigenvector of A

II. If **p** and **q** are eigenvectors corresponding to eigenvalue λ of **A**, then so is **p + q** as long as $\mathbf{p} + \mathbf{q} \neq \mathbf{0}$.

III. If λ is an eigenvalue of **A** with corresponding eigenvector **x**, then for any positive integer r, λ^r is an eigenvalue of $\mathbf{A}^r$ with corresponding eigenvector **x.**

IV. I If λ and μ are an eigenvalues of **A** with $\lambda \neq \mu$ and **p** is an eigenvector corresponding to λ and **q** is an eigenvector corresponding to μ, then **p** and **q** are linearly independent. Property IV is often stated as: Eigenvectors corresponding to distinct eigenvalues of the same matrix are linearly independent. This implies that if all the eigenvalues of an n × n matrix are different (distinct), then we have n linearly independent eigenvectors associated with the matrix

V. A and $\mathbf{A}^T$ have the same eigenvalues.

VI. If **A** is diagonal, upper triangular, or lower triangular, then its eigenvalues are its diagonal entries.

VII. The eigenvalues of a symmetric matrix are real numbers.

VIII. det(A) is the product of the eigenvalues of **A**.

IX. **A** is nonsingular if and only if 0 is not an eigenvalue of **A**, or equivalently, **A** is singular if and only if 0 is an eigenvalue of **A**.

X. The eigenspace associated with an eigenvalue $\lambda$of matrix **A** is the set of all eigenvectors corresponding to the eigenvalue together with the zero vector.

Definition A matrix **B** is said to be **similar** to a matrix **A** if there is a nonsingular matrix **P** such that $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$.

XI. Similar matrices have the same eigenvalues. (But not necessarily the same eigenvectors.)

XII. An n × n matrix **A** is diagonalizable if and only if it has n linearly independent eigenvectors.

Definition A matrix is said to be **diagonalizable** if it is similar to a diagonal matrix.

XIII. If an n × n matrix **A** has distinct eigenvalues, then it has n linearly independent eigenvectors (see IV.) so it is similar to a diagonal matrix.

Definition: A matrix **A** is called **defective** if it has a repeated eigenvalue (multiplicity m > 1) for which there are fewer linearly independent eigenvectors than the number of repeats.

XIV. Symmetric matrices are guaranteed to be similar to a diagonal matrix.