

Eigen Exercises

(NAME)

1. If matrix A is 2 by 2 and has eigenvalues $\lambda = 1$ and $\lambda = -2$, what are the eigenvalues of A^3 ?
Answer: _____

2. The characteristic polynomial of matrix A is $\lambda^4 - 3\lambda^2 - 4$.
(a) What size is matrix A? _____
(b) What are the eigenvalues of A? _____
(c) How many linearly independent eigenvectors does matrix A have? _____

3. The characteristic polynomial of matrix A is $\lambda^3 + 4\lambda^2 + 4\lambda$.
(a) What size is matrix A? _____
(b) What are the eigenvalues of A? _____
(c) Is it guaranteed that A has 3 linearly independent eigenvectors?
Circle the appropriate response: YES NO
Briefly explain why your answer is correct: _____

4. Find a basis for the eigenspace of $A = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ corresponding to eigenvalue 4.

Basis: _____ What is the dimension of this eigenspace? _____

5. Find a basis for the eigenspace of $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ corresponding to eigenvalue 4.

Basis: _____ What is the dimension of this eigenspace? _____

6. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) $\lambda = 3$ is an eigenvalue of A. Find a corresponding eigenvector. Answer: _____
(b) Without doing any computations name an eigenvalue of A different from $\lambda = 3$.
Answer: _____ Give a reason that your answer is correct: _____

7. The eigenvalues of A are 2, -3, 6, and 1. What is $\det(A)$? Answer: _____

8. Let $A = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}$. One eigenvalue of A is $\lambda = 4$.

(a) Find an eigenvector of A corresponding to $\lambda = 4$. Answer: _____

(b) Find an eigenvector of A^T corresponding to $\lambda = 4$. Answer: _____

9. Construct a matrix whose characteristic polynomial is $\lambda^3 - 3\lambda^2 + 2\lambda$.

Answer: _____

10. If square matrix A has a row of zeros, what is one eigenvalue of A? Answer: _____

Give a reason that your answer is correct: _____

11. If square matrix A has a column of zeros, what is one eigenvalue of A? Answer: _____

Give a reason that your answer is correct: _____

12. A is a square matrix such that the sum of the entries in each row is the number k. Prove that k is an eigenvalue of A.

Put your Proof here:

13. A is a square matrix such that the sum of the entries in each column is the number r. Prove that r is an eigenvalue of A.

Put your Proof here:

14. Let S be the set of all 3 by 3 diagonal matrices that have eigenvalue $\lambda = 7$. Is S a subspace of M_{33} ? Explain.

15. Let A be a 3 by 3 diagonal matrix such that $\lim_{k \rightarrow \infty} A^k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Describe the set of eigenvalues of A.

Answer: _____