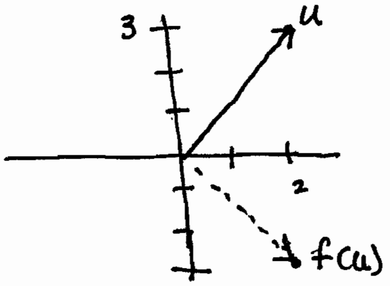
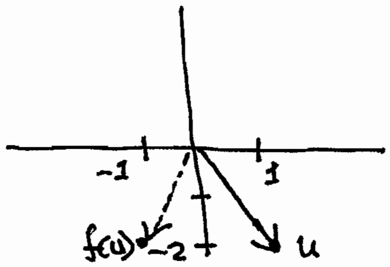
Section 1.5 Assigned Problems

1.



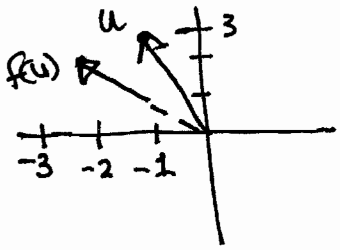
$$f(u) = Au = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

2.



$$f(u) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

3.



$$f(u) = \begin{bmatrix} \cos(30^\circ) & -\sin(30^\circ) \\ \sin(30^\circ) & \cos(30^\circ) \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -\sqrt{3}/2 - \frac{3}{2} \\ -1/2 + 3\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} \frac{-3-\sqrt{3}}{2} \\ \frac{3\sqrt{3}-1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -2.36603 \\ 2.09808 \end{bmatrix}$$

9. We seek vector x (if there is one) so that $Ax = w$

$$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \Rightarrow \begin{array}{l} x + 3y = 7 \\ -x + 2y \;\; 3 \end{array}$$ Solve the system

Add eqn #1 to eqn #2 $\Rightarrow$ $\begin{array}{l} x + 3y = 7 \\ 5y = 10 \end{array} \Leftrightarrow$ So $y = 2$

Then $x + 3y = x + 6 = 7 \Rightarrow x = 1$

This says, there is a vector so that $A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = w$,

hence w is in the range.

10. We seek a vector x (if there is one) so that $Ax = w$.

$$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \Rightarrow \begin{array}{l} x + 3y = 4 \\ -x + 2y = 1 \end{array}$$ Solve the system.

Add eqn #1 to eqn #2 $\Rightarrow$ $\begin{array}{l} x + 3y = 4 \\ 5y = 5 \end{array} \Leftrightarrow y = 1$

Then $x + 3y = x + 3 = 4 \Rightarrow x = 1$

This says, there is a vector so that $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = w$,

hence w is in the range.

15. (a) Reflection about the y-axis; see #2.

(b) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$; Note if $\theta = \pi/2$ then $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

is this matrix; so rotation $\pi/2$ radians counterclockwise.

17. (a) Projection onto x-axis

(b) Projection onto y-axis

19. (a) Rotation $60°$ counterclockwise (See T10 Sec. 1.4)

(c) 12 ; $360°/30°$.

T1. $f(u) = Au$

(a) Show $f(u+v) = f(u) + f(v)$

$f(u+v) = A(u+v) = Au + Av$ (Prop. of matrix products)

$= f(u) + f(v)$ (Def. of f.)

(b) Show $f(cu) = c\,f(u)$

$f(cu) = A(cu) = c(Au)$ (Prop of scalar mult.)

$= c\,(f(u))$ (Def. of f.)

(c) Show $f(cu+dv) = c\,f(u) + d\,f(v)$

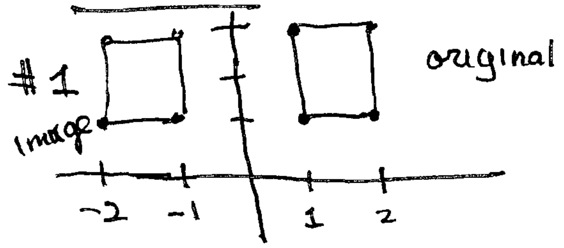
$f(cu+dv) = A(cu+dv) = A(cu) + A(dv)$ (by prop matrix mult)

$= c(Au) + d(Av)$ (by prop scalar mult.)

$= c\,f(u) + d\,f(v)$ (def. of f)

Section 2.3

#1



$$A\begin{bmatrix}1&2&1&2\\1&1&3&3\end{bmatrix}=\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}1&2&1&2\\1&1&3&3\end{bmatrix}$$
$$=\begin{bmatrix}-1&-2&-1&-2\\1&1&3&3\end{bmatrix}$$

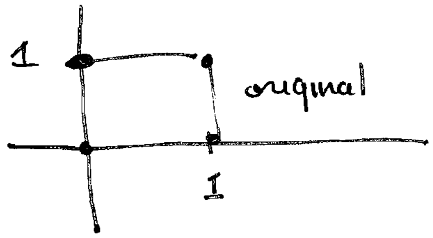
#8

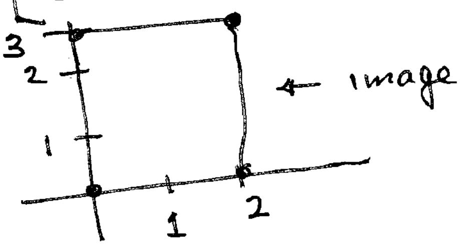
$$\begin{bmatrix}4&-3\\-4&2\end{bmatrix}\begin{bmatrix}1&-3&2\\1&-3&-1\end{bmatrix}=\begin{bmatrix}1&-3&11\\-2&6&-10\end{bmatrix}$$

#12

$A=\begin{bmatrix}2&0\\0&3\end{bmatrix}$ from Example #5

$$A\begin{bmatrix}0&1&1&0\\0&0&1&1\end{bmatrix}=\begin{bmatrix}0&2&2&0\\0&0&3&3\end{bmatrix}$$





#. ML.2

(a) 8 (b) 1 (c) $BA=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

This is the identity matrix.

BA * coord of unit square

= coord. of unit square