Solutions to Assigned Exercises in Section 1.6 Part 2

#21(a)
The augmented matrix is

1	1	2	3	|	13
1	-2	1	1	|	8
3	1	1	-1	|	1

Used row operations **-1 * Row 1 + Row 2** and **-3 * Row 1 + Row 3** to get

1	1	2	3	|	13
0	-3	-1	-2	|	-5
0	-2	-5	-10	|	-38

Used row operation **-1/3 * Row 2** to get

1.0000	1.0000	2.0000	3.0000	13.0000
0	1.0000	0.3333	0.6667	1.6667
0	-2.0000	-5.0000	-10.0000	-38.0000

Or equivalently in terms of fractions

1	1	2	3	|	13
0	1	1/3	2/3	|	5/3
0	-2	-5	-10	|	-38

Used row operation **2 * Row 2 + Row 3** to get

1	1	2	3	|	13
0	1	1/3	2/3	|	5/3
0	0	-13/3	-26/3	|	-104/3

Used row operation **-3/13 * Row 3** to get

1	1	2	3	|	13
0	1	1/3	2/3	|	5/3
0	0	1	2	|	8

This matrix is in REF

Used row operation **-1/3 * Row 3 + Row 2** to get

1	1	2	3	|	13
0	1	0	0	|	-1
0	0	1	2	|	8

Used row operation **-2 * Row 3 + Row 1** to get

1	1	0	-1	|	-3
0	1	0	0	|	-1
0	0	1	2	|	8

Used row operation **-1 * Row 2 + Row 1** to get

1	0	0	-1	|	-2
0	1	0	0	|	-1
0	0	1	2	|	8

This matrix is in RREF

To fund the solution use back substitution; note that we have 3 equations in 4 unknowns so there will be one variable that can be chosen arbitrarily (often called a free variable). The system is

$$\begin{aligned} \mathbf{x} \quad\quad - \mathbf{w} &= -2 \\ \mathbf{y} \quad\quad &= -1 \\ \mathbf{z} + 2\mathbf{w} &= 8 \end{aligned}$$

In the third equation solve for z, $\mathbf{z} = 8 - 2\mathbf{w}$; next we have $\mathbf{y} = -1$ and $\mathbf{x} = -2 + \mathbf{w}$. If we let w be the arbitrary constant r we have

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \\ \mathbf{w} \end{bmatrix} = \begin{bmatrix} -2+\mathbf{w} \\ -1 \\ 8-2\mathbf{w} \\ \mathbf{w} \end{bmatrix} = \begin{bmatrix} -2+\mathbf{r} \\ -1 \\ 8-2\mathbf{r} \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 8 \\ 0 \end{bmatrix} + \mathbf{r} \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}.$$

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#21(b)

The augmented matrix is

1	1	1	|	1
1	1	-2	|	3
2	1	1	|	2

Used row operations **-1 * Row 1 + Row 2** and **-2 * Row 1 + Row 3** to get

1	1	1	|	1
0	0	-3	|	2
0	-1	-1	|	0

Used row operation **Row 2 <--> Row 3** to get

1	1	1	|	1
0	-1	-1	|	0
0	0	-3	|	2

Used row operations **-1 * Row 2** and **-1/3 * Row 3** to get

1.0000	1.0000	1.0000	|	1.0000
0	1.0000	1.0000	|	0
0	0	1.0000	|	-0.6667

This matrix is in REF

Or equivalently in terms of fractions

1	1	1	|	1
0	1	1	|	0
0	0	1	|	-2/3

This matrix is in REF

Used row operations **-1 * Row 3 + Row 2** and **-1 * Row 3 + Row 1** to get

1	1	0	|	5/3
0	1	0	|	2/3
0	0	1	|	-2/3

Used row operation **-1 * Row 2 + Row 1** to get

1	0	0	|	1
0	1	0	|	2/3
0	0	1	|	-2/3

This matrix is in RREF

Since the coefficient matrix part of the RREF is the 3 by 3 identity matrix this system of 3 equations in 3 unknowns has a unique solution

x = 1, y = 2/3, and z = -2/3

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#22(a)

The augmented matrix is

2	-1	1	|	3
1	-3	1	|	4
-5	0	-2	|	-5

Used row operation **Row 1 <--> Row 2** to get
(this gives us a leading 1 in row 1)

1	-3	1	|	4
2	-1	1	|	3
-5	0	-2	|	-5

Used row operations **-2 * Row 1 + Row 2** and **5 * Row 1 + Row 3** to get

1	-3	1	|	4
0	5	-1	|	-5
0	-15	3	|	15

Used row operation **<u>3 * Row 2 + Row 3</u>** to get

1	-3	1	4
0	5	-1	-5
0	0	0	0

Here we can observe we have 2 equations in 3 unknowns so will have one arbitrary constant (or free variable).

Used row operation 1/5 * Row 2 to get

1.0000	-3.0000	1.0000	|	4.0000
0	1.0000	-0.2000	|	-1.0000
0	0	0	|	0

Or equivalently in fractions

1	-3	1	|	4
0	1	-1/5	|	-1
0	0	0	|	0

This matrix is in REF

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Used row operation **3 * Row 2 + Row 1** to get

1	0	2/5	1
0	1	-1/5	-1
0	0	0	0

This matrix is in RREF

Solving the system

$$\begin{aligned} \mathbf{x} \quad + 2/5\mathbf{z} &= 1 \\ \mathbf{y} - 1/5\mathbf{z} &= -1 \end{aligned}$$

we get **y = -1 + 1/5 z, x = 1 – 2/5 z, and z = r, an arbitrary constant. The solution is**

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} 1 - 2/5\mathbf{z} \\ -1 + 1/5\mathbf{z} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \mathbf{r} \begin{bmatrix} -2/5 \\ 1/5 \\ 1 \end{bmatrix}$$

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MY PROBLEM

$$\begin{aligned} \mathbf{x} + \mathbf{y} + 2\mathbf{z} &= -1 \\ \mathbf{x} - 2\mathbf{y} + \mathbf{z} &= -5 \\ 3\mathbf{x} \quad + 5\mathbf{z} &= 0 \end{aligned}$$

The augmented matrix is

$$\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 0 & 5 & 0 \end{array}$$

Used row operations **-1 * Row 1 + Row 2** and **-3 * Row 1 + Row 3** to get

$$\begin{array}{cccc} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 0 & -3 & -1 & 3 \end{array}$$

Used row operation -1 * Row 2 + Row 3 to get

$$\begin{array}{cccc} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 0 & 0 & 0 & 7 \end{array}$$

Inspecting the last row we see the system is inconsistent.

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#23

The augmented matrix is $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & \mathbf{a}^2-5 & \mathbf{a} \end{array}\right]$. Apply row operations $-1R_1 + R_2$, $-1R_1 + R_3$ to get equivalent system $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & \mathbf{a}^2-4 & \mathbf{a}-2 \end{array}\right]$.

(a) The system will have no solution if the last row is [0 0 0 | *] where * ≠ 0. This will happen if a = -2.

(b) There will be a unique solution provided we can chose a so that the (3,3)-entry is not zero. (Because then we could get a leading 1 in each row.) So if a ≠ 2 and a ≠ -2 we will have a unique solution.

(c) There will be infinitely many solutions if the last row is all zeros. This will happen if a = 2.

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#31 Solve the linear system $\left[\begin{array}{ccc|c} 4 & 1 & 3 & 4 \\ 2 & -1 & 3 & 5 \\ 2 & 2 & 0 & -1 \end{array}\right]$. Its RREF is

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3/2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ so there are infinitely many vectors that will work. **x = 3/2 – z, y = -2 + z, z = r**

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#35 Use row operations to obtain the RREF of the coefficient matrix part of the augmented matrix with two augmented columns: $\left[\begin{array}{cc|c|c} 1 & -1 & 1 & 5 \\ 2 & 3 & -8 & -5 \end{array}\right]$. We get $\left[\begin{array}{cc|c|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -2 & -3 \end{array}\right]$. So the solution of the first system is x = -1, y = -2 and the solution of the second system is x = 2, y = -3.

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#37 The augmented matrix of the homogeneous system is $\left[\begin{array}{ccc|c} -5 & 0 & -5 & 0 \\ -1 & -5 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]$ and its RREF is $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

Then x = -z, y = 0, z = z so a nontrivial solution is any vector of the form $\begin{bmatrix} -\mathbf{r} \\ 0 \\ \mathbf{r} \end{bmatrix}$.

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#47 Use equation (16) to get the system of equations

$$\begin{bmatrix} 1 & 1 & 1 \\ 9 & 3 & 1 \\ 25 & 5 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{a}_2 \\ \mathbf{a}_1 \\ \mathbf{a}_0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}.$$

Solving this system we get **$\mathbf{a_2}$ = ½, $\mathbf{a_1}$ = -3/2 $\mathbf{a_0}$ = 3**.

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#53 $p(x) = ax^2 + bx + c$, $p'(x) = 2ax + b$, $p''(x) = 2a$
$f(x) = e^{2x}$ so $f'(x) = 2e^{2x}$, $f''(x) = 4e^{2x}$

p(0) = f(0) ➔ c = 1,
p'(0) = f'(0) ➔ b = 2,
p''(0) = f''(0) ➔ 2a = 4