Solutions to assigned problems for Section 1.7

5(a)

A =

1	3
-2	6

Append 2 by 2 identity to A and apply row operations.

1	3	1	0
-2	6	0	1

Use row operation **2 * Row 1 + Row 2** to get

1	3	1	0
0	12	2	1

Use row operation **1/12 * Row 2** to get

1	3	1	0
0	1	1/6	1/12

Use row operation **-3 * Row 2 + Row 1** to get

1	0	1/2	-1/4
0	1	1/6	1/12

A^{-1} =

1/2	**-1/4**
1/6	**1/12**

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5(b)

A =

1	2	3
1	1	2
0	1	2

Append 3 by 3 identity to A and apply row operations.

1	2	3	1	0	0
1	1	2	0	1	0
0	1	2	0	0	1

Apply row operation **-1 * Row 1 + Row 2** to get

1	2	3	1	0	0
0	-1	-1	-1	1	0
0	1	2	0	0	1

Apply row operation **-1 * Row 2** to get

1	2	3	1	0	0
0	1	1	1	-1	0
0	1	2	0	0	1

Apply row operation **-1 * Row 2 + Row 3** to get

1	2	3	1	0	0
0	1	1	1	-1	0
0	0	1	-1	1	1

Apply row operation **-1 * Row 3 + Row 2** to get

1	2	3	1	0	0
0	1	0	2	-2	-1
0	0	1	-1	1	1

Apply row operation **-3 * Row 3 + Row 1** to get

1	2	0	4	-3	-3
0	1	0	2	-2	-1
0	0	1	-1	1	1

Apply row operation **-2 * Row 2 + Row 1** to get

1	0	0	0	1	-1
0	1	0	2	-2	-1
0	0	1	-1	1	1

A^{-1} =

0	**1**	**-1**
2	**-2**	**-1**
-1	**1**	**1**

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6(a)
A =

1	3
2	6

Append 2 by 2 identity to A and apply row operations.

1	3	1	0
2	6	0	1

Apply row operation -2 * Row 1 + Row 2 to get

1	3	1	0
0	0	-2	1

Since the coefficient matrix part has a zero row there is NO WAY to use row operations to get a 2 by 2 identity in the first two columns, so the matrix A has no inverse.

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6(b)

A =

1	2	3
0	2	3
1	2	4

Append 3 by 3 identity to A and apply row operations.

1	2	3	1	0	0
0	2	3	0	1	0
1	2	4	0	0	1

Apply row operation **-1 * Row 1 + Row 3** to get

1	2	3	1	0	0
0	2	3	0	1	0
0	0	1	-1	0	1

Apply row operation **-3 * Row 3 + Row 2** to get

1	2	3	1	0	0
0	2	0	3	1	-3
0	0	1	-1	0	1

Apply row operation -3 * Row 3 + Row 1 to get

1	2	0	4	0	-3
0	2	0	3	1	-3
0	0	1	-1	0	1

Apply row operation -1 * Row 2 + Row 1 to get

1	0	0	1	-1	0
0	2	0	3	1	-3
0	0	1	-1	0	1

Apply row operation **1/2 * Row 2** to get

The current matrix is:

1	0	0	1	-1	0
0	1	0	3/2	1/2	-3/2
0	0	1	-1	0	1

A^{-1} =

1	**-1**	**0**
3/2	**1/2**	**-3/2**
-1	**0**	**1**

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11. Strategy find the RREF of the augmented matrices.

(a) The RREF is

1	0	1	|	0
0	1	1	|	0
0	0	0	|	0

so we have 2 equations in 3 unknowns so there is a nontrivial solution.

Note that the first & third equations of the original system are the same, so we will get a zero row in the RREF.

(b) The RREF is

1	0	-1	|	0
0	1	1	|	0
0	0	0	|	0

so we have 2 equations in 3 unknowns so there is a nontrivial solution.

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13. Find the inverse of A^{-1}. We get

$$\begin{matrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{matrix}$$

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15. A matrix is nonsingular if it is row equivalent to an identity matrix. This implies that the RREF of a nonsingular matrix must be and identity matrix. IF a matrix has a row or column of all zeros, then no matter what row operations we use we can not get its RREF to be an identity matrix. (A row of zeros or a column of zeros will remain in the RREF of a marix.)

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20. (a) This is not true. Take $A = I_2$ an $B = -I_2$; both of these matrices are nonsingular (Why?). Their sum is the 2 by 2 zero matrix which can't be nonsingular by Exercise 15.

(b) This is true since $(cA)(1/cA^{-1}) = I$.

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22. If A is nonsingular, then –A is also nonsingular by using #20(b). The other two parts are not true by #20 (a).

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23. $\mathbf{D}^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$ To see this compute RREF 0f $[D \mid I_3]$.

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24. Since $(AB)^{-1} = B^{-1}A^{-1}$, just compute the product $B^{-1}A^{-1}$ which gives

$$\begin{matrix} 11 & 19 \\ 7 & 0 \end{matrix}$$

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25. Just compute the product $A^{-1}b$ which gives $\begin{bmatrix} 19 \\ 23 \end{bmatrix}$.

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26. Since the homogeneous system has a nontrivial solution the coefficient matrix must be singular.

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T1. AB = O and B is nonsingular so B-1 exist. Multiply both sides of AB = O by B^{-1} on the right; $ABB^{-1}=OB^{-1}$ ➔ A = O.

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T8. A is symmetric so $A = A^T$. To show A^{-1} is symmetric we show that $(A^{-1})^T = A^{-1}$. We do this as follows: $(A^{-1})^T = (A^T)^{-1}$ by Thm 1.10(c)

$= A^{-1}$ since $A = A^T$