<u>**Review Problems for Final in Linear Algebra Fall 2009**</u>

1. Let $u = [1 \quad 2 \quad 3 \quad 0]$ and $v = [2 \quad -1 \quad 0 \quad 1]$.

(a) Find a nonzero vector w that is orthogonal to both u and v.
(b) How many such vectors w are there? Explain

2. Let **A** be a 6 × 6 matrix with row 6 equal to the sum of the first 5 rows. For each of the following topics discuss properties of matrix **A**. {det, rank, rref, singular/nonsingular, eigenvalues, diagonalizable, null space, row space, defective}

3. Let $\mathbf{A} = \begin{bmatrix} a & 0 & b & c \\ 0 & k & 0 & 0 \\ d & 0 & e & f \\ g & 0 & h & m \end{bmatrix}$. Show that k is an eigenvalue of **A**.

4. System Ax = b has $\text{rref}([A\,|\,b]) = \left[\begin{array}{cccc|c} 1 & -2 & 0 & 4 & 5 \\ 0 & 0 & 1 & 7 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. Find a basis for the null space of **A**.

5. Let $\mathbf{A} = \begin{bmatrix} 1 & x & 1 \\ 0 & -1 & x \\ 2 & 2 & 1 \end{bmatrix}$. (a) Determine all values of x so that det(**A**) = 1. (b) Find the value of x where det(**A**) is a minimum. (c) For x a real number, can **A** ever be a singular matrix?

6. Explain how to construct a matrix **A** with nonzero entries so that its eigenvalues are 0, 1, 2.

7. Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$ and vectors $u = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$. Show that u and v are eigenvectors of **A** and find the corresponding eigenvalues.

8. Which of the following sets span R^3? (a) $S = \{[2 \quad 1 \quad 0], [-2 \quad 1 \quad 1], [4 \quad 0 \quad -1]\}$.

(b) $T = \{[1 \quad 2 \quad 3], [3 \quad 0 \quad 1], [2 \quad 2 \quad 2], [6 \quad 4 \quad 6]\}$.

9. Determine which of the following are subspaces.

(a) S = all continuous functions f(x) so that f(5) = 6.

(b) W = all 3 × 3 matrices **A** such that $\mathbf{A}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 7\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(c) W = all 2 × 3 matrices **A** whose null space includes the vector $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

(d) W = all 3 × 1 matrices x so that the dot product of x and $[1 \quad 2 \quad 5]$ is zero.

10. Let W be the subspace of 3 × 3 matrices with 2nd row and 2nd column all zero. Find a basis for W.