**Review problems for Second Exam Fall 2009**

Chapter 3 Determinants ➔ P. 213 In the Chapter Test #1,2,4,6
➔ P. 212 In the supplementary problems #4,16

Chapter 4. Vectors in $R^n$ ➔ Chapter test #1-6

Chapter 5. Only the material on cross products ➔ P. 263 #1b,c , 2d

Chapter 6. Real vector spaces ➔ P.278 #4, 15 (just check if the sets are closed)
➔ P. 288 #16 b, c, 17b, 27b
➔ P. 301 #3a,d ,7, 9, 11a,b (just determine if LI or LD), 15
➔ P. 374 in the Chapter Test 1-3, 6a,b,c,d,h,j

Note: 1. Solution to questions in the Chapter Tests are in the back of the book.

2. You have done lots of homework; the solutions to assigned problems are on my web site.

3. **You should be able to give definitions, describe various ideas, describe how to solve particular types of problems, describe how various ideas are related to one another, and of course solve problems like on the homework.**

The format of exam will be similar to that of the first exam.

Determinant and its properties

Vectors: dot product, length, angle between, orthogonal vectors, parallel vectors, unit vector, linear transformations (image, range, standard matrix representing a linear transformation)

Cross products

Vector space, subspace, closure, span, linear independence/dependence, solution space of $Ax = 0$ (also called the null space of A), row space of A, the column space of A, basis, dimension of a vector space (or subspace), rank of matrix.

Connections of topics to singular matrices and connections of topics to nonsingular matrices.

Name 5 “different ways” to determine if a matrix is nonsingular.

Determinants

1. Compute the determinant of each of the following and state whether the matrix is singular or nonsingular.

(a) $A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -1 & 3 \\ 1 & -2 & 2 \end{bmatrix}$ (b) $B = \begin{bmatrix} 2 & -1 & 3 & 2 \\ -1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 1 \\ 0 & 0 & 5 & -2 \end{bmatrix}$

2. Find all values of x so that matrix $A = \begin{bmatrix} 1 & 1 & x \\ 1 & x & x \\ x & x & x \end{bmatrix}$ is singular.

3. If A and S are n × n matrices and S is nonsingular, prove that $\det\left(S^{-1}AS\right) = \det(A)$. (Show your work.)

4. Find all values of x so that $\det\left(xI_2 - A\right) = 0$ given that $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

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Vectors

1. Determine the set of <u>all</u> vectors [a b c d] which are orthogonal to $v = \begin{bmatrix} 1 & 2 & 0 & -1 \end{bmatrix}$. (Hint: Use dot products to get one equation in four unknowns and use it to determine the form of vectors orthogonal to v.)

2. Find the cosine of the angle between $v = \begin{bmatrix} 1 & -3 & 2 & 4 \end{bmatrix}$ and $w = \begin{bmatrix} 3 & 1 & 0 & 2 \end{bmatrix}$.

3. Find a unit vector parallel to $v = \begin{bmatrix} 2 & -2 & 1 & -1 \end{bmatrix}$.

4. Show that the set S of all vectors orthogonal to a fixed vector v is a closed set.

5. Determine the cross product of $v = 3i + 4j - 2k$ and $w = -i + 2j + 3k$.

6. Let L: $R^3 \to R^4$ defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2y - z \\ 3x + y + z \\ 0 \end{bmatrix}$. Show that L is a linear transformation by expressing it as a matrix transformation. Determine if $v = \begin{bmatrix} -1 \\ 6 \\ 3 \\ 0 \end{bmatrix}$ is in the range of L.

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Closure, Span, Subspaces, Linear Independence/Dependence, Bases, Dimension, Rank

1. Let b, c, and d be any real numbers and a = -2b + d. The set S of vectors of the form $\begin{bmatrix} -2b+d \\ b \\ c \\ d \end{bmatrix}$ is a subspace in $R^4$. Find a basis for this subspace and state its dimension.

2. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Define S to be the set of all 2 × 2 matrices B such that AB = BA. Determine if S is a subspace of $M_{22}$ .

3. Find a basis for the subspace of all 2 × 2 lower triangular matrices and state its dimension.

4. Does the set of vectors $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix} \right\}$ span $R^3$? (Show your work.)

5. Let $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$. Show that S is a basis for $R^3$, then express $v = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ as a linear combination of the vectors in S.

6. Is $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ a basis for $R^4$?

7. Find a basis for the row space of $A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 7 & 1 & 8 \end{bmatrix}$. Find the rank of A. Without doing any additional computations determine the dimension of the null space of A.

8. The RREF of matrix A is $\begin{bmatrix} 1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Find a basis for the null space of A.

9. A nonhomogeneous linear system Ax = b has its set of all solutions expressed in the following form where r, s, and t are free variables.

$\begin{bmatrix} r \\ r+2s-9 \\ s \\ s-t+6 \\ t \end{bmatrix}$. Express the solution set as the sum of the set of solutions to the corresponding homogeneous system Ax = 0 denoted $x_h$ and a particular solution of Ax = b denoted $x_p$.