SOLUTIONS/Answers

Review 1.1 – 1.7; TRUE/ FALSE[1]

T 1. If $\mathbf{A} = \mathbf{A}^T$, then **A** is symmetric. T 2. $(\mathbf{A} - \mathbf{B})^T = \mathbf{A}^T - \mathbf{B}^T$ T 3. Matrix addition is commutative.

F 4. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is a submatrix of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. T 5. $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$.

F 6. Every diagonal matrix is a scalar multiple of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

T 7. Let S be the set of all matrices of the form $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$, then every vector in S is a linear combination of S = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

T 8. If **x** and **y** are both 5 by 1 matrices, then $\mathbf{x} \bullet \mathbf{y}$ is the same as $\mathbf{x}^T\mathbf{y}$.

T 9. If **A** is 3 by 4 and **x** is 4 by 1, then **Ax** is a linear combination of the columns of **A**.

F 10. The matrix transformation **f(c) = Ac** for $\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ projects vector **c** onto the x-axis.

T 11. A homogeneous linear system is guaranteed to be consistent.

F 12. It is possible for a linear system of equations to have exactly two different solutions.

F 13. The linear system with augmented matrix $\left[\begin{array}{cccc|c} 1 & -2 & 3 & 8 & 1 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ has one degree of freedom.

F 14. The linear system with coefficient matrix

$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 4 \\ -2 & 1 & 1 \end{bmatrix}$ and right side $\mathbf{b} = \begin{bmatrix} 5 \\ 6 \\ -3 \end{bmatrix}$ has solution $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

T 15. If **A** is nonsingular then RREF of **A** is an identity matrix.

T 16. If **A** is nonsingular then **Ax = b** has a unique solution.

F 17. A homogeneous system with a square coefficient matrix always has only the trivial solution.

F 18. The inverse of a product of matrices is the product of the inverses; that is, $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$.

T 19. The RREF of a matrix is unique.

[1] Review1_1to1_7.doc \LINEAR ALGEBRA FALL 2007

Exercises

1. Find t so that the dot product of vectors $\begin{bmatrix} t & 2 & 1 & 3 \end{bmatrix}$ and $\begin{bmatrix} -1 & 1 & 3 & t \end{bmatrix}$ is zero. $t = -5/2$

2. Find all solutions to the linear system $\begin{aligned} x &+y &-z &= 5 \\ 2x &+y &+z &= 2 \\ x &-y &-2z &= 3 \end{aligned}$. only soln $\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

3. Find all solutions to the linear system $\begin{aligned} x &+2y & &+w &= 0 \\ -3x &-6y &-3z &+3w &= 0 \\ 2x &+4y &+z & &= 0 \end{aligned}$. $r\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$

4. If possible, find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$

5. Let $A = \begin{bmatrix} 5 & 3 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ and t = 4. Find all solutions to the homogeneous system $(tI_3 - A)\mathbf{x} = \mathbf{0}$. $r\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$

6. The trace of a square matrix A, denoted tr(A) is the sum of its diagonal entries.
   (a) Explain why $\text{tr}(A) = \text{tr}(A^T)$. $\text{ent}_{ii}(A) = \text{ent}_{ii}(A^T)$
   (b) If B is the same size as A, show that tr(A + B) = tr(A) + tr(B). Use properties of addition and def. of the trace
   $\text{tr}(A+B) = \sum_{j=1}^{n}(a_{jj}+b_{jj}) = \sum a_{jj} + \sum b_{jj} = \text{tr}(A) + \text{tr}(B)$

7. If A, B, and C are $n \times n$ and $ABC = I_n$. State a formula for A^{-1}. $A^{-1} = BC$

8. Matrix A is diagonal. When is A guaranteed to be nonsingular? $\text{ent}_{ii}(A) \neq 0$ for all i

9. Given $A^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$. Solve linear system Ax = b. $x = \begin{bmatrix} -4 \\ 14 \\ 25 \end{bmatrix}$

10. Linear system Ax = b is such that the reduced row echelon form of [A } b] is

$$\begin{array}{ccccc|c} x & y & z & w & v & \\ \end{array}$$
$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -6 & 12 \\ 0 & 1 & -3 & 0 & 7 & -18 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} 12-2z+6v \\ -18+3z-7v \\ z \\ 4 \\ v \end{bmatrix} = \begin{bmatrix} 12 \\ -18 \\ 0 \\ 4 \\ 0 \end{bmatrix} + z\begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + v\begin{bmatrix} 6 \\ -7 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

. Write the solution as a linear combination of columns.

11. Matrix transformation f: $R^3 \to R^3$ is defined by f(x) = Ax. See if Ax=b is consistent.
    (a) Given a vector b in R^3, how do you determine if b is in the range of f?
    (b) Show that if A is nonsingular, then every vector b in R^3 is in the range of f.
    Ax=b for any b is consistent.