Span Exercise Solutions

Exercises All work must be shown.

In Exercises 1-3, determine if **v** is in span(S).

1. $\mathbf{v} = \begin{bmatrix} 9 \\ 10 \\ -1 \end{bmatrix}$, $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \right\}$. Form the linear combination with unknown of the members of S and set it equal to v. The corresponding linear system has augmented matrix

$$\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 0 & 3 & 10 \\ 0 & 1 & -1 & -1 \end{array}$$

its RREF is

$$\boxed{\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}}$$

The system is consistent so v is in span(S).

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2. $\mathbf{v} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $S = \left\{ \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \right\}$. Form the linear combination with unknown of the members of S and set it equal to v. The corresponding linear system has augmented matrix

$$\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 \end{array}$$

its RREF is

$$\boxed{\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}}$$

The system is inconsistent so v is not in span(S).

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3. $\mathbf{v} = 6t^2 + 9t - 3$, $S = \{t + 1, t^2 - 2, t^2 + 2t\}$. Form the linear combination with unknown of the members of S and set it equal to v. The corresponding linear system has augmented matrix

$$\begin{array}{ccc|c} 0 & 1 & 1 & 6 \\ 1 & 0 & 2 & 9 \\ 1 & -2 & 0 & -3 \end{array}$$

its RREF is

$$\boxed{\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}}$$

The system is consistent so v is in span(S).

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In Exercies 4-7, give a concise description of the span(S).

4. $S = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. **Span(S) = all vectors in R^2 with the same two entries.**

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5. $S = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \right\}$. **Span(S) = all 3 by 3 matrices first first and second rows all zeros.**

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6. $S = \{t^3, t^2, t, 1\}$. **Span(S) = all polynomials of degree 3 or less.**

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7. $S = \{i, j, k\}$. **Span(S) = R^3.**

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In Exercises 8-10, determine if span(S) = W.

8. $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, W = $\mathbf{R}^3$. Let $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be arbitrary member of R^3. We form a linear combination of the members of S with unknown coefficients and set it equal to v. We then compute the rref of the corresponding augmented matrix.

```
1  0  1  1 | a
0  1  0  1 | b   has RREF
1  2  2  1 | c
```

```
1    0    0    3  | 2*a+2*b-c
0    1    0    1  |     b
0    0    1   -2  | -2*b-a+c
```

Since the system is consistent for all choices of a, b, and c span(S) = W.

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9. $S = \left\{ \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\}$, W = M_{22}. Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be arbitrary member of M_{22}. We form a linear combination of the members of S with unknown coefficients and set it equal to v. We then compute the rref of the corresponding augmented matrix.

```
0  0  1  1  | a
2  0  0  0  | c  has RREF
0  1  0  1  | b
1  1  0  1  |d
```

```
1     0     0     0  |     1/2*c
0     1     0     1  |         b
0     0     1     1  |         a
0     0     0     0  |-b-1/2*c+d
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The system is inconsistent if -b -1/2*c+d is not zero, so span(S) is not W.

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10. $S = \{t+1, t+2, t^2\}$, W = P_2. Let $at^2 + bt + c$ be arbitrary member of P_2. We form a linear combination of the members of S with unknown coefficients and set it equal to v. We then compute the rref of the corresponding augmented matrix.

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & a \\ 1 & 1 & 0 & b \\ 1 & 2 & 0 & c \end{array}\right] \text{ has RREF } \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2*b-c \\ 0 & 1 & 0 & -b+c \\ 0 & 0 & 1 & a \end{array}\right]$$

Since the system is consistent for all choices of a, b, and c span(S) = W.

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In Exercises 11 and 12, find a spanning set for the set of all solutions to the homogeneous systems.

11. **Ax = 0**, where $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$. Form the augmented matrix [A **|** 0], compute its rref, and form the general solution. RREF of [A **|** 0] is

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

We have $x_1 = -x_3$, $x_2 = -x_3$, $x_4 = 0$
So the general solution is

$$\mathbf{x} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$ hence **a spanning set for the solution set is** $\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

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12. **Ax = 0** where $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Form the augmented matrix [A **|** 0], compute its rref, and form the general solution. RREF of [A **|** 0] is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

So the only solution is the trivial solution hence a spanning set is the zero vector.