

What can you do with LINEAR ALGEBRA?¹

This question is easier to answer than ‘What is linear algebra?’ since to describe an area of mathematics to someone who does not know the jargon tends to cause frustration and is often counter productive. [So rather than list technical terms associated with linear algebra we choose to illustrate some topics that can modeled with the tools of linear algebra.](#)

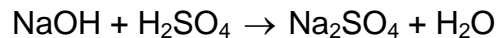
Constructing Equations:

- Given three distinct points, find a parabola that goes through the points.
- Given three distinct points that lie on a circle, find the radius and center of the circle.
- Determine the orbit of an asteroid about the sun. (The orbit must be an ellipse by Kepler’s first law. The equation of an ellipse is $Ax^2+Bxy+Cy^2+Dy+E=0$.)

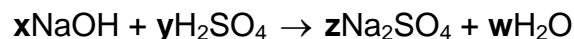
Strategy: Set up a Cartesian coordinate system in the plane of the orbit with the sun at the origin. Then make five observations of the position of the asteroid in this system at five different times. Use the set of five points together with the equation of the ellipse to determine coefficients A, B, C, D, and E.

Chemistry:

- Balancing a Chemical Equation (Application of the Law of Conservation of Mass)
For example, sodium hydroxide (NaOH) reacts with sulfuric acid (H₂SO₄) to form sodium sulfate (Na₂SO₄) and water H₂O. The chemical equation is expressed as

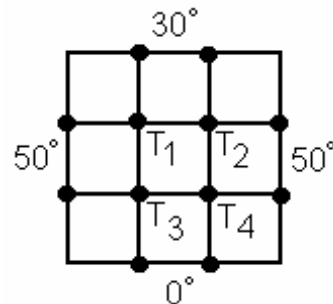


Balancing this chemical reaction means finding values of x, y, z and w so that the number of atoms of each element is the same on both sides of



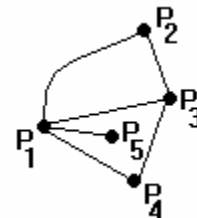
Temperature Distribution:

- A simple model for estimating the temperature distribution on a square plate where the edges are held fixed at certain temperatures involves imposing a grid on the plate and then estimating the temperatures at the grid points.



Graph Theory:

- By a **graph** we mean a set of points called **nodes** or **vertices**, some of which are connected by **edges**. The nodes are usually labeled as P₁, P₂, ..., P_k and we often allow an edge to be traveled in either direction. Develop a model that computes the number of paths of length 3 from P₁ to P₄.



¹ What can you do with Linear Algebra.doc D.R. Hill, Temple University, 8/4/2006

Facial Recognition:

- Photographs of a face can be represented mathematically, stored, and then quickly reconstructed using mathematical models that process the stored data. This application is part of complex systems involving facial recognition and locating a face within an image or video clip. (Over half of our cerebral cortex is involved in visual recognition, more than is used for mathematics.)

Markov Chains:

- Suppose that you have a physical process with n possible states and at any particular time it is in one of those states. Furthermore suppose that the form at the k^{th} state depends only on the form at the previous state, the $k-1^{\text{st}}$ state. Such a process is called a Markov Chain or Markov Process. To build a mathematical description of such a process we use linear algebra together with probability.

Economics:

- In order to understand and be able to manipulate the economy of a country or a region, one needs to come up with a certain model based on the various sectors of this economy. The **Leontief model** is an attempt in this direction. Based on the assumption that each industry in the economy has two types of demands: external demand (from outside the system) and internal demand (demand placed on one industry by another in the same system), the Leontief model represents the economy as a system of linear equations. (The Leontief model was invented in the 30's by Professor Wassily Leontief (picture above) who developed an economic model of the United States economy by dividing it into 500 economic sectors. On October 18, 1973, Professor Leontief was awarded the Nobel Prize in economy for his effort.)

Global Positioning System:

- A Global Positioning System (GPS) is a satellite based global navigation system enabling the user to determine his or her position in 3-dimensional coordinates without the need for further knowledge of navigational calculations. It was developed by the military as a locating utility and the GPS system operated by the U. S. Department of Defense became operational in 1995. GPS technology has proven to be a useful tool for a wide variety of civilian applications as well, and is now available in low-cost units. These units have been incorporated into boats, automobiles, and airplanes, in addition to hand-held units for general use such as hiking. GPS is based on satellite ranging; that is, calculating the distances between a receiver and the position of 3 or more satellites (4 or more if elevation is desired) and then applying mathematics.

Data Modeling:

- The purpose of data modeling is to develop an accurate model, or graphical representation of a client's information. The data model acts as a framework for the development of the new or enhanced application. One basic such modeling process is called the **line of best fit**; here we seek the equation of a straight line that in some sense comes closest to all points in a 2-dimensional data set, but need not go through any of those points.

Other areas include Genetics, **Animation**, Cryptography, Coding, Image Compression, Video Games, Simulations, etc.