Assignments for Basis, etc.

Section 6.4

#2. Note $\dim(R^3) = 3$

(a) Only 2 vectors – can't be a basis for R^3

(b) Four vectors – all bases for R^3 have 3 vectors – so this can't be a basis

(c) Must check. – Since we have 3 vectors we can use Thm 6.9 and check to see if the 3 vectors are L. I.

Find rref of $\left[\begin{array}{ccc|c} 3 & -1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 2 & 1 & 0 & 0 \end{array}\right] \Rightarrow$ get $\left[\begin{array}{c|c} I_3 & \begin{matrix}0\\0\\0\end{matrix} \end{array}\right]$ so they

are LI & since we have 3 vectors ($= \dim R^3$) they are a basis.

#6 Note $\dim(M_{22}) = 4$.

Again we can use Thm 6.9 & see if these vectors are LI.

Find rref of $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \Rightarrow$ get $\left[\begin{array}{c|c} I_4 & \begin{matrix}0\\0\\0\\0\end{matrix} \end{array}\right]$ so they

are LI & since we have 4 vectors they are a bases.

#11 Follow the technique in Example 5.

Find RREF of $\left[\begin{array}{cccc|c} 1 & 3 & 11 & 7 & 0 \\ 2 & 2 & 10 & 6 & 0 \\ 2 & 1 & 7 & 4 & 0 \end{array}\right] \Rightarrow$ Get $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

The leading 1's point to $v_1 = (1,2,2)$ and $v_2 = (3,2,1)$

So $\{v_1, v_2\}$ is a basis for span (S).

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#8(b) Again $\dim \mathbb{R}^3 = 3 \Rightarrow$ every basis for $\mathbb{R}^3$ has 3 vectors
Here we have 3 vectors so we can use Thm 6.9 & just
check to see if they are L I.

Find RREF $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 2 & 4 & 0 \\ 2 & 0 & -1 & 0 \end{array}\right] \Rightarrow$ Get $\left[\begin{array}{c|c} I_3 & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{array}\right]$

So this set is a basis for $\mathbb{R}^3$

#12 Follow the technique of Example 5.

Find RREF of $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & -6 & 0 \\ -1 & 1 & -1 & -3 & 0 \end{array}\right] \Rightarrow$ Get $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

So the leading 1's point to v_1, v_2, v_3, thus
$\{v_1, v_2, v_3\}$ is a basis for span(S).

#17a (a, b, c) where $b = a + c \Rightarrow (a, a+c, c)$
$= a(1, 1, 0) + c(0, 1, 1)$

We have that $\{(1,1,0), (0,1,1)\}$ is a spanning set.
They are also L I (check it!) $\Rightarrow$ they are a basis.

#18(a) $(0, b, c) = b(0,1,0) + c(0,0,1)$ + you can check
$S = \{(0,1,0), (0,0,1)\}$ is a spanning set
that they are L I $\Rightarrow S$ is a basis.

(b) (a,b,c) with $a - b + 5c = 0 \Rightarrow a = b - 5c$
So $(a,b,c) = (b-5c, b, c) = b(1,1,0) + c(-5,0,1)$
$S = \{(1,1,0), (-5,0,1)\}$ spans & this pair of vectors
can be shown to be L I $\Rightarrow$ S is a basis

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#19 (b) $(a, b, c, d) = (a, b, a-b, a+b)$
$= a(1, 0, 1, 1) + b(0, 1, -1, 1)$

$S = \{(1, 0, 1, 1), (0, 1, -1, 1)\}$ spans & can be shown to be LI $\Rightarrow$ S is a basis

#20 (a) $(a, b, c, d) = (b, b, c, d) = b(1, 1, 0, 0) + c(0, 0, 1, 0) + d(0, 0, 0, 1)$

$S = \{(1, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$

spans — Just check that they are LI (& they are)

So S is a basis

#31 $\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} = a\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$+ d\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + e\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + f\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The six matrices span the 3x3 symmetric matrices
Show they are LI (and they are), so we have a basis

#32 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Similar reasoning as in #31

#33 Lots of answers: For example — all vectors of the form $(a, b, 0, 0)$ a, b any real numbers

Section 6.6

#5 (a) Find RREF of A - the nonzero rows will be ~~as~~ basis for row space of A that are not rows of A ; RREF is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

So $\{(1,0,-1), (0,1,0)\}$ is a basis for row(A)

#6 (a) Use the technique as in #5 ; RREF is $\begin{bmatrix} 1 & 0 & 0 & -33/7 \\ 0 & 1 & 0 & 23/7 \\ 0 & 0 & 1 & -8/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

First 3 rows are a basis for row(A)

#13 compute RREF of $[A|0]$.

We get $\left[\begin{array}{cccc|c} 1 & 0 & -3 & -4\frac{1}{3} & 0 \\ 0 & 1 & 2 & 3\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

rank(A) = 2

dim nullspace = 2

(2 free variables)

#14 As in #13 RREF of $[A|0]$ is $[I_4|0]$

rank(A) = 4 ; dim nullspace = 0