Linear Algebra Fall 2009

Assignments for Chapter 3, Determinants

Notation: Our text often uses the notation $\begin{vmatrix} 5 & 3 & 2 \\ 1 & -1 & 4 \\ 0 & 7 & 8 \end{vmatrix}$ to indicate that you are to compute the determinant of the matrix $\begin{bmatrix} 5 & 3 & 2 \\ 1 & -1 & 4 \\ 0 & 7 & 8 \end{bmatrix}$. ↑↑↑↑↑ The notation above looks like absolute value bars around the matrix. I will most often write **det** $\left(\begin{bmatrix} 5 & 3 & 2 \\ 1 & -1 & 4 \\ 0 & 7 & 8 \end{bmatrix}\right)$ to indicate the determinant. (Both notations are common; it's a matter of personal preference.

Section 3.1 Page 193
Use our "devices" for 2 × 2 and 3 × 3 matrices to compute the determinants in Exercises #5(a), (b) 6(a), (b), (c), 11,12,13,14,17(a), 18(b), (c), 20(a) Show your work.
NOTE: in #11 and 12 the answer is a polynomial in λ. Simplify the answer as much as possible.

Section 3.1 Page 193
Use properties or rules for determinants of special matrices to compute the determinants in Exercises #5(c), 15 (a), (b), 16(a), 18(a), 20(b) State the property used and of course the value of the determinant.

Section 3.1 Solve Exercises T5, T6, T8, T9, T10 Give reasons for the steps you use to show the statements.

Section 3.2 Page 208 Use the method of determinants by expansion to solve #3(b), 4(a), 6(a). Show your work.

Section 3.1 Page 194 Use the method of row operation (also called reduction to upper triangular form) to find the determinant in 17(a), (b), 18(b), (c), 20(c) & solve #22, 23. Show your work.

Properties of the Determinant

$\mathbf{det(AB) = det(A)\ det(B) = det(BA)}$ $\mathbf{det(A)} \neq \mathbf{0}$ if and only if **A** is **nonsingular**. $\mathbf{det(A) = 0}$ if and only if **A** is **singular**. $\mathbf{det(A^{-1})} = \dfrac{\mathbf{1}}{\mathbf{det(A)}}$ $\mathbf{det(I_n) = 1}$ **det(**diagonal matrix**)** = product of its diagonal entries **det(**upper triangular matrix**)** = product of its diagonal entries **det(**lower triangular matrix**)** = product of its diagonal entries $\mathbf{det(A^T) = det(A)}$ $\mathbf{det(}k\mathbf{A)} = k^n\ \mathbf{det(A)}$, if **A** is $n \times n$ $\mathbf{det(A_{kR_i})} = k\,\mathbf{det(A)}$ $\mathbf{det(A_{R_i \leftrightarrow R_j})} = -\mathbf{det(A)}$, $i \neq j$ $\mathbf{det(A_{kR_i + R_j})} = \mathbf{det(A)}$
If two rows (columns) of **A** are equal, then **det(A)** = 0.
If two rows (columns) of **A** consist entirely of zeros, then **det(A)** = 0.

Table 1.

<u>Equivalent Statements for an $n \times n$ Nonsingular Matrix A</u>

1. **A** is nonsingular.
2. $\mathbf{rref(A) = I_n}$.
3. **A** has an inverse.
4. **Ax = b** has a unique solution for every right side **b**.
5. **Ax = 0** has only the trivial solution.
6. **det(A)** $\neq 0$.

Observation: For an $n \times n$ matrix **A**, the homogeneous system **Ax = 0** has a nontrivial solution if and only if **det(A)** = 0 (that is, **A** is singular).