Vector Space Exercises

Section 6.2

#1 Yes, it is closed under addition & scalar mult. Note the line consists of all Points (x,x) x any real #. (1^{st} & 2^{nd} coord are equal)

$(t,t)+(s,s) = (s+t, s+t)$

$k(t,t) = (kt, kt)$

#2 Yes, it is closed under addition & scalar multiplication. Adding $(x_1,y_1,0)+(x_2,y_2,0)$ gives an ordered triple with 3^{rd} coord. $=0$.

$k(x_1,y_1,0) = (kx_1, ky_1, 0)$

#3 No – If (x,y) is in or on the circle, then $k(x,y)$ need not be in or on the circle; for example $10(\frac{1}{2},\frac{1}{2}) = (5,5)$ ← Not in or on the circle. Can also check that closure of addition fails.

#4 No – as in answer to #3 $10(\frac{1}{2},\frac{1}{2}) = (5,5)$ ← not in the unit square. Can also check that closure of addition fails.

#5 (a) No – not closed under addition. Also $(0,0,0)$ not in the set.

(b) Vectors look like $(a,b,a+b) = a(1,0,1)+b(0,1,1)$ which is the span$\{(1,0,1),(0,1,1)\}$ which is a subspace. Or just check closures.

(c) No – zero is not in the set

#6 (b) Vectors look like $(-c,b,c) = c(-1,0,1)+b(0,1,0)$

This is the span of $\{(-1,0,1),(0,1,0)\}$

Also just check closures; they both hold.

(c) Vectors look like $(a, 2a+1, c)$. The zero vector is not in the set. Note if $a=c=0$, then the vector is $(0,1,0)$, not $(0,0,0)$.

Also can show not closed.

Section 6.2

#7a. No! The vectors have the form (a, b, c, d) with $a-b=2 \Rightarrow a=b+2$.
So the vector really look like $(b+2, b, c, d)$. These will not be closed under addition or scalar multiplication.

#8(b) No! Here the vectors will have the form $(1, 0, 1-d, d)$.
This set will not be closed under addition.

(c) No. This set of vectors can't contain the zero vector.

#14 $W = \text{span}\{(1,2,-3), (-2,3,0)\}$ By Thm 6.3 W is a subspace.

#18 (a) Yes — adding 2 $n\times n$ symmetric matrices gives a symmetric matrix; a scalar times a symmetric matrix is also a symmetric matrix.
So we have closures.

(b) No — Not closed under scalar mult & $A=0$ ← singular
" " " addition $I_n + (-I_n) = 0$

(c) Yes — sum of diagonal matrices is a diagonal matrix
& scalar times a diagonal matrix is a diagonal matrix

#25 (a) Find RREF $\left[\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 2 & 2 \\ 1 & 0 & 1 & 2 \end{array}\right]$ It is $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Which says the system is inconsistent. Hence v is <u>not</u> in span $\{v_1, v_2, v_3\}$.

#26 (b) Find RREF $\left[\begin{array}{ccc|c} 1 & 1 & 2 & -3 \\ 0 & 0 & -1 & 3 \\ -1 & 1 & 2 & -1 \\ 3 & 2 & 1 & 2 \end{array}\right]$ It is $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

System is consistent — So Yes it is in the span.

Sec. 6.2

\# T3 Let $Ay = b$ and $Az = b$. Check closures.

Consider $A(y+z) = Ay + Az = b + b = 2b$.

So $y+z$ is NOT a soln $\Rightarrow$ set of all solns of $Ax=b$ $(b \neq 0)$ is not a subspace.

\# T6 null space of A = set of all solns to $Ax=0$.

If A is nonsingular, then A^{-1} exists. Then

$$A^{-1}Ax = A^{-1}0 \Rightarrow x = 0$$

So null space of $A = \{0\}$ ← only zero vector.

Q1. Lots of answers are possible; one ans. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Q2. $A = I_3$ — But A = any nonsingular matrix also will work; $\text{rref}(A^T) = I_3$.

Q3. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is one answer; lots of others can be used, but the rref must look like

Q4. One answer is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. But lots of others are possible as $(\text{rref}(A^T))^T$ looks like