Tool Kit Assignments

Read Pages 54-68.

Exercises for **Section 2.1**: #1a, b, 2, 13, 15, 18	**Bisection**

For #1 and #2 do the computations by hand/calculator showing your steps to 5 decimal places. Record values in a table like

Left End Point XL Right End Point XR Midpoints XM Function Value at XM

Indicate the signs of the function values at the ends of the intervals.

For #13, 15, 18 use software.

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Section 6.2 Numerical Differentiation Read pages 438 - 445

Exercises are listed next.

#1. Derive the coefficients in the approximation

$$f'(x) \cong a_0 f(x-h) + a_1 f(x + h/3) + a_2 f(x+h)$$

Choose coefficients so that the approximation is exact for all polynomials of degree two. Establish the order of convergence for this approximation.

#2. Let $f(x) = \begin{cases} 1, & x \le 1 \\ 0, & x > 1 \end{cases}$. Make the approximations requested in the following table. Then discuss the numerical results.

$$D_1(h) = \frac{f(x_0 + h) - f(x_0)}{h}$$

$$D_2(h) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

Approximating $f'(1)$.

h	$D_1(h)$	$D_2(h)$
0.1		
0.001		
0.001		

#3. Centered differences use data both to the left and right of x_0 to estimate $f'(x_0)$. At times it is useful to have $O\left(h^2\right)$ approximations that use data just on one side of x_0. The following pair of formulas do just that.

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)}{2h} + O\left(h^2\right) \qquad \textbf{(RF)}$$

$$f'(x_0) = \frac{f(x_0 - 2h) - 4f(x_0 - h) + 3f(x_0)}{2h} + O\left(h^2\right) \qquad \textbf{(LF)}$$

For the data in the following table use formulas RF or LF to estimate $f'(x)$. For each value of x indicate which formula you used.

x	f(x)	Estimate of f '(x)	(RF or LF)
2	9.20765095258201		
2.2	10.8037055878372		
2.4	12.6442880139644		
2.6	14.8040416017375		
2.8	17.3826135915336		
3	20.5088969473673		

#4. For any reasonable formula that estimates f '(x) as a linear combination of function values of f like

$$f'(x) \approx \sum_{k=1}^{n} A_k f(x_k)$$

Explain why $\sum_{k=1}^{n} A_k = 0$.

#5. In a circuit with impressed voltage E(t) and inductance L, Kirchoff's first Law gives the relationship $E(t) = L\frac{di}{dt} + Ri$ where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain

t	1.00	1.01	1.02	1.03	1.04
i	3.10	3.12	3.14	3.18	3.24

Where t is measured in seconds, I is in amperes, the inductance L is a constant 0.98 henries, and the resistance is 0.142 ohms. Approximate the voltage E at the values of t given in the table. Use second order formulas. (Show your work including the formula used for each value of t.)