Bradie_Exercises_Section 1.1

EXERCISES Section 1.1

1. Use the statistics algorithm from the text to compute the mean, $\bar{x}$, and the standard deviation, s, of the data set: $-5, -3, 2, -2, 1$.

2. With $n = 4$, use the trapezoidal rule algorithm from the text to approximate the value of the definite integral

$$\int_0^1 \frac{1}{1+x^2}\,dx.$$

3. Use the square root algorithm from the text to approximate $\sqrt{5}$. Take $x_0 = 5$ $\epsilon = 5 \times 10^{-4}$ and $Nmax = 10$.

4. A different scheme for approximating the square root of a positive real number is based on the recursive formula

$$x_{n+1} = \frac{x_n^3 + 3x_n a}{3x_n^2 + a}.$$

(a) Construct an algorithm for approximating the square root of a given positive real number a using this formula.

(b) Test your algorithm using $a = 2$ and $x_0 = 2$. Allow a maximum of 10 iterations and use a convergence tolerance of $\epsilon = 5 \times 10^{-5}$. Compare the performance of this algorithm with the one presented in the text.

6. Consider the computation of the following sum, $\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n} a_i b_j$,

where the a_i and b_j are real numbers.

(a) How many multiplications and how many additions are required to compute the sum? Each answer should be a function of n.

(b) Modify the summation to an equivalent form that reduces the number of operations needed. How many multiplications and how many additions are required to compute the sum in its revised form?

For Exercises 15–18, make use of the fact that when the sum of a convergent alternating series is approximated using the sum of the first n terms, the error in this approximation is smaller than the magnitude of the $(n+1)$st term; that is, if $\sum(-1)^n a_n$ is an alternating series with sum S, then

$$\left| S - \sum_{k=0}^{n-1} (-1)^k a_k \right| < a_n.$$

15. The value of π is given by $\displaystyle \pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \cdots\right).$

(a) Construct an algorithm to approximate the value of π to within a specified tolerance, ϵ.

(b) Test your algorithm with a tolerance value of $\epsilon = 5 \times 10^{-3}$.