Linear Interpolation Exercises

#1. Do Exercise 14(a) on Page 352.

#2. Table Construction

Purpose: Develop an appropriate strategy for constructing a table of values of **f(x)** = $\sqrt{\mathbf{x}}$ over [1, 2] so that linear interpolation can be used to estimate $\sqrt{\mathbf{t}}$ for non-tabulated values **t** with an **absolute error** $\le$ **$5*10^{-6}$.**

Situation: We will assume that the number of accurate digits in the data that will be recorded in the table is 7 or more (if needed). Hence all errors are due to linear interpolation of existing accurate data. This shifts the burden of construction so that we need to determine the **spacing h** between successive members of the table. Naturally we want **h** to be as large as possible so that the table can be as short as possible.

Assumptions and Notation:

h > 0; $x_i = 1 + i*h$ and the table will have entries x_i and $f(x_i)$; i = 0, 1, 2, … , N, where N is the smallest integer greater than $\frac{2-1}{h}$. If t is in $[x_i, x_{i+1}]$, then

$$\mathbf{f(t) \approx P_1(t) =} \text{ interpolating polynomial of degree 1 at the two points } \{ (x_i, f(x_i)), (x_{i+1}, f(x_{i+1})) \};$$

(a) Determine the largest value of **h** so that **E(h)**, the error in linear interpolation, $\le 5*10^{-6}$. What is the integer value of N? (The value of **h** should be rounded down to have 3 decimal places.)

(b) Make a list of the first 5 entries of the table; you can use your calculator or MATLAB to obtain this data.

(c) To check things out, use linear interpolation to estimate the square root at the midpoint of the third and fourth entries of the table in part (b). Compute the absolute error in this estimate.

3. Do Exercise 10 on Page 351. (Hint: it is important here that you realize that $g(x) = (x - x_0)(x - x_1)$ is a parabola so it is “easy” to determine the maximum $|g(x)|$ over the interval between x_0 and x_1.)