

# **INTRODUCTION<sup>1</sup>**

## **Questions:**

**What do we mean by "Numerical Methods and Analysis"?**

**What kind of mathematics is this course & book about?**

**Generally and broadly speaking; this course & book covers the mathematics and methodologies that underlie the techniques of **scientific computation or computational mathematics**.**

**Example: Consider the button on your calculator that computes the sine of the number in the display. Exactly how does the calculator know that "correct" value?**

**When we speak of using the computer to solve a complicated mathematics or engineering problem, exactly what is involved in making that happen?**

- **Computers are not "born" with the knowledge of how to solve complicated mathematical and engineering problems. They are **programmed** to do it, and **the programs implement algorithms that are based on the kinds of things we will talk about in this course.****

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<sup>1</sup> NA\_general\_intro.doc DRH \numanal\_fall\_2009 6/12/2009

Textbooks and courses in this area generally follow one of two main themes.

- "**Numerical methods**" which tend to emphasize the implementation of the algorithms, perhaps at the expense of the underlying mathematical theory that explains why the methods work.
- "**Numerical analysis**" tends to emphasize this underlying mathematical theory, perhaps at the expense of some of the implementation issues.

**Our approach is to properly mix the study of the algorithms and their implementation ("methods") with the study of the mathematical theory ("analysis") that supports them. That is the goal of the text and this course.**

In today's world numerical analysis can best be described as the '**study of algorithms for problems of continuous mathematics**'. Numerical analysis supplies techniques to areas of scientific computation that provide rapid convergence of sequential approximations to a wide variety of problems:

*As such it uses computing to develop insight into the facets of a problem, rather than just produce numbers associated with the approximation.*

Numerical analysis is widely used by practitioners in diverse fields of science, engineering, and business. It appears in so many forms that at times it seems to be the *invisible glue* that holds together a set of procedures that provide a realistic model of complex problems. Given this behind the scene role, numerical analysis is widely regarded as an unglamorous subject. But in truth, **numerical analysis is a rich blending of algorithms, mathematics, and computing** that plays a pivotal role in our world. In many ways the study of numerical analysis provides a training ground for analytically thinking in the area of computational algorithms.



There are a number of different ways to break down the subject into component parts. We will discuss

- The derivation of the algorithms,
- The implementation of the algorithms, and
- The mathematical analysis of algorithms, in order to learn how best to use them and how best to implement them.

In our study of each technique, **we will usually be concerned with two issues that often are competing with each other:**

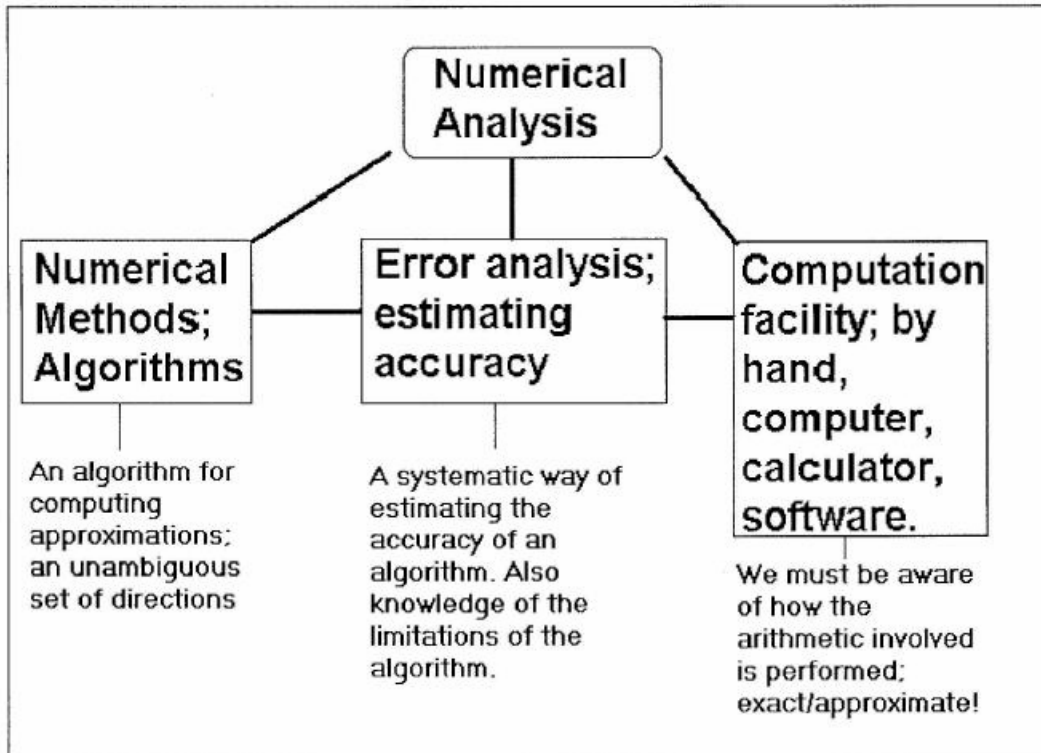
- **Accuracy:** Very few of our computations will yield the exact answer to the problem, so we will have to understand how much error is made, and how to control (or even diminish) that error.
- **Efficiency:** Does the algorithm take an inordinate amount of computer time? This might seem to be an odd question to concern us with - after all, computers are fast, right? - But there are slow ways and fast ways to do things. All else being equal (it rarely is), we prefer the fast ways.

We say these two issues compete with each other because, generally speaking, the steps that can be taken to make an algorithm more accurate usually make it more costly, i.e., less efficient.

There is a third issue of importance.

- **Stability:** Does the method produce similar results for similar data? If we change the data by a small amount, do we get vastly different results? If so, we say the method is unstable, and unstable methods tend to produce unreliable results. It is entirely possible to have an accurate method that is efficiently implemented, and yet is horribly unstable.

Numerical Analysis is concerned with the mathematical derivation, description, and analysis of methods for obtaining numerical solutions of mathematical problems.



## MOTIVATION FOR USING COMPUTERS

1. Reduce costs
2. Relieve the boredom & errors inherent in human calculators
3. Increase the speed
4. The complexity of the calculations needed in many areas of science, engineering, business, medicine, & finance is beyond the reasonable expectation of "by hand" work.

**"The purpose of computing is insight, not numbers.",**  
R.W.Hamming

**It is easy to become entranced by the overt power of computers, and thereby to abuse their application.**  
Weizenbaum --Computer Power and Human Reason

**How do we use numerical analysis?** Some scenarios.

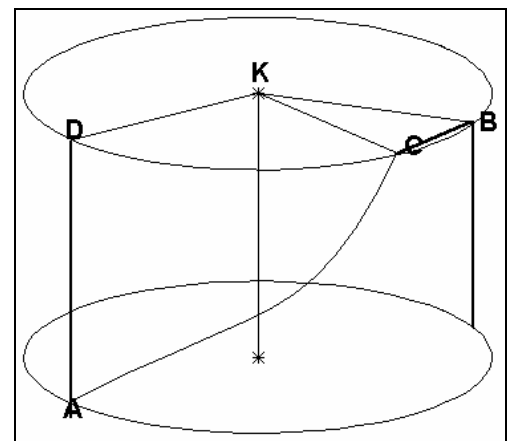
**Trigonometric Functions:**

a) When you press the sine button on a calculator, what is done to return the numerical value?

b) When you use the sine function in MATLAB, what is done to return the numerical value? (Similar questions apply to the other trigonometric functions as well as exp and log.)

**Distance between points on the top and bottom rims of a cylinder.**

Point B is on the top rim of the cylinder and point A on the bottom rim. Our objective is to determine the length of the shortest path from B to A. There are many paths from B to A. One goes directly from B to A adhering to the curved face of the cylinder. For any point C between D and B another path goes directly from B to C then goes down to A adhering to the curved face of the cylinder. One such path is shown in the figure.



**Tank Calibration:**

Consider a cylindrical tank is lying on its side, as on a tank truck. The basic problem is to calibrate a dipstick so that a volume of fluid in the tank can be read from the dipstick markings. The difficulty comes from the type of equipment that may be in the tank. For instance tanks mounted on moving vehicles often contain baffles to prevent instability due to sloshing of the fluid and/or pumping equipment whose shape and volume are not accurately known.

**In the Windows utility PAINT, how does the button that draws a curve work?**



**How do laser printers draw all those fonts?**

## Global Positioning System:

- A Global Positioning System (GPS) is a satellite based global navigation system enabling the user to determine his or her position in 3-dimensional coordinates without the need for further knowledge of navigational calculations. It was developed by the military as a locating utility and the GPS system operated by the U. S. Department of Defense became operational in 1995. GPS technology has proven to be a useful tool for a wide variety of civilian applications as well, and is now available in low-cost units. These units have been incorporated into boats, automobiles, and airplanes, in addition to hand-held units for general use such as hiking. GPS is based on satellite ranging; that is, calculating the distances between a receiver and the position of 3 or more satellites (4 or more if elevation is desired) and then applying mathematics.

## Area of a Golf Green:



## Volume of a Light Bulb:



## Volume of a Sea Worm:

