Trapezoidal Rule Exercises

1. Graph y = f(x) over [a, b] and rotate the resulting curve about the x-axis. The surface area of the corresponding surface of revolution is given by

$$\textbf{surface area} = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\ dx$$

Use the MATLAB routine **trap** to approximate the surface area using the composite trapezoidal rule n subintervals (n is the number of applications of the standard trapezoidal rule that are applied to the subintervals).

(a) $f(x) = x^2$, over [0, 1] True value ≈ 3.8097297048578
(b) f(x) = sin(x), over [0, π/4] True value ≈ 2.42242805082268
(c) f(x) = cos(x), over [0, π/4] True value ≈ 4.78937167338436
(d) $f(x) = e^x$, over [0, 1] (In MATLAB e^x is coded as exp(x).)
True value ≈ 22.9430223411722

Using n = 10, 20, 40, 80, construct a table with the following information.

n	Trap Approx.	e_n = Absolute Error	e_n/e_{2n}

Display the approximations in a table to at least 10 decimal places. If the last two digits of your Temple ID are

00 – 30, do part (a)
31 – 50, do part (b)
51 – 70, do part (c)
71 – 99, do part (d)

2. Since the composite trapezoidal rule in $O(h^2)$, we expect that the when n is doubled that the ratios of $\frac{e_n}{e_{2n}}$ should approach 4. When the integral $\int_0^1 \left(1+\sin(\sqrt{x})\right) dx$ is approximated by the composite trapezoidal rule the following information is obtained. Explain why the ratios $\frac{e_n}{e_{2n}}$ do not approach 4. (The exact value of the integral is about 1.60233735787951.)

n	Trap. Approx	e_n =Absolute Error	e_n/e_{2n}
1	1.42073549240395	-0.181601865475565	
2	1.53518621574201	-0.0671511421375079	2.70437493235323
4	1.57788948787613	-0.0244478700033877	2.74670726440394
8	1.59352511833545	-0.00881223954406352	2.77430837883399
16	1.59918121821022	-0.00315613966929029	2.79209428841439
32	1.60121165589387	-0.00112570198564099	2.80370800580327
64	1.60193695595661	-0.000400401922908245	2.8114300187788
128	1.60219520235516	-0.000142155524351706	2.81664694168067
256	1.60228695197246	-5.0405907058515e-005	2.82021557883446
512	1.60231950041192	-1.78574675921972e-005	2.82267946439056
1024	1.60233103529168	-6.32258783683781e-006	2.82439217184976

3. We know that Distance Traveled from time = a to time = b is given by $\int_{a}^{b}$ **velocity dt**.

We have the data in the table given next. **Explain** how to use the trapezoidal rule to approximate the distance travel from t = 1 to t = 10.

t = time in minutes	1	2	3.5	5	6.5	7	9	9.5	10
v = velocity in meters/minute	4	6	5.5	6	8	8.5	7	7.5	6