

SPLINE DEFINITIONS¹

Definition 1. A function $S(x)$ is called a **spline of degree k** if

1. The domain of $S(x)$ is an interval $[a, b]$.
2. $S, S', S'', \dots, S^{(k-1)}$ are all continuous on $[a, b]$.
3. There are points x_i (called the **knots** of S) such that $a = x_0 < x_1 < x_2 < \dots < x_n = b$ and **S is a polynomial of degree k on each subinterval $[x_i, x_{i+1}]$.**

So S is a piecewise polynomial of degree k which is continuous and has $k-1$ continuous derivatives in $[a, b]$.

(The continuity conditions need only be checked at the interior knots since elsewhere we have the same polynomial on both sides of a point or we use one-sided continuity at $x = a$ and $x = b$.)

Definition 2. A **cubic spline** is a spline of degree 3. On the interval $[x_i, x_{i+1}]$ is a cubic polynomial of the form $a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$ $i = 0, 1, 2, \dots, n-1$.

Definition 3. An **interpolatory cubic spline** for ordered data set $\{(x_i, f(x_i)) \mid i = 0, 1, 2, \dots, n\}$ is a cubic spline $S(x)$ such that the knots the knots are x_i and

$$S(x_i) = f(x_i), \quad i = 0, 1, 2, \dots, n.$$

Definition 4. An **interpolatory natural cubic spline** is an interpolatory cubic spline that satisfies

$$S''(x_0) = 0 \text{ and } S''(x_n) = 0.$$

(We often just say **natural cubic spline** or **free spline**.)

Definition 5. An **interpolatory clamped cubic spline** is an interpolatory cubic spline that satisfies

$$S'(x_0) = \alpha \text{ and } S'(x_n) = \beta$$

where α and β are specified values. (That is, the first derivatives at the ends are specified values. We often just say **clamped cubic spline**.)

Definition 6. An **interpolatory not-a-knot cubic spline** is an interpolatory cubic spline which requires that **S''' be continuous at $x = x_1$ and $x = x_{n-1}$** . (We often just say not-a-knot cubic spline; this just means we have no information other than the values of f at each knot.)

NOTES: 1. There are many other types of (interpolatory) cubic splines.

2. A quadratic spline is a spline of degree 2. There is definition for interpolatory quadratic splines that corresponds to Definition 3.

¹ Splinedefs.doc \numanal_fall_2009 DRH 8/13/09